

# Stored Value Cards: Costly Private Substitutes for Government Currency

---

Jeffrey M. Lacker

Stored value cards look like credit cards but are capable of storing monetary value. There are a number of stored value card systems being developed in the United States, and some have already been implemented in Europe and elsewhere. Stored value cards are particularly well-suited for transactions that would otherwise be carried out with currency and thus are a private substitute for government fiat money, like private bank notes. Unlike bank notes, however, stored value cards employ new technologies that are quite different from, and potentially more costly than, the coins and paper currency they are aimed at replacing. This article explores the basic welfare economics of costly private substitutes for government currency, an important class of payments system innovations.<sup>1</sup>

Consumers and merchants are likely to benefit from the introduction of stored value cards. Many might prefer to avoid the inconvenience and cost of handling paper currency. The usual presumption, in the absence of market imperfection, is that a successful new product must provide social benefits in excess of social costs. Issuers will attempt to cover their costs and earn a competitive return while providing consumers and merchants with a means of payment they prefer. They can do so if consumers and merchants are collectively willing to pay enough, either directly or indirectly, to remunerate

---

■ The author would like to thank Michelle Kezar and Helen Upton for research assistance, reviewers Mike Dotsey, Tom Humphrey, Ned Prescott, and John Weinberg for extensive comments, James McAfee for useful references, Urs Birchler, Scott Freeman, Marvin Goodfriend, Don Hester, Dave Humphrey, Peter Ireland, Bob King, Bennett McCallum, Geoffrey Miller, Kevin Reffett, Will Roberds, Stacey Schreft, and Bruce Summers for helpful comments and conversations, and an anonymous reviewer for helpful comments. The author is solely responsible for the contents of this article. The views expressed do not necessarily reflect those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

<sup>1</sup> I will use the term “currency” rather than “currency and coin” to refer to government-supplied notes and coinage; the analysis applies equally to government-minted coin.

issuers for the opportunity cost of the inputs devoted to the alternative means of payment. If stored value thrives, standard reasoning suggests that it must be because the value to consumers and merchants exceeds the cost of provision.

The fact that stored value is a monetary asset provides further reason to believe that it will be beneficial. Currency is subject to an implicit tax due to inflation, which reduces its rate of return relative to other risk-free nominal assets. Like any other (non-lump-sum) tax, the inflation tax distorts economic decisions, giving rise to deadweight costs as people try to economize on the use of currency. Private substitutes for currency provide a means of avoiding the seigniorage tax, alleviating the deadweight loss associated with any given inflation rate. Stored value cards can increase economic welfare by easing the burden of inflation.

Stored value cards could be socially wasteful, however. Stored value liabilities compete with an asset, currency, that pays no interest, while issuers are free to invest in interest-earning assets. Thus one portion of the return to stored value issuers is the spread between market interest rates and the rate of return on currency. This return far exceeds the government's cost of producing and maintaining the supply of currency—less than two-tenths of a cent per year per dollar of currency outstanding.<sup>2</sup> At current interest rates the private incentive to provide stored value exceeds the social cost of the currency replaced by as much as 4 or 5 cents per dollar. Thus stored value cards, if successful, will replace virtually costless government currency with a substitute that could cost substantially more.

This article presents a model in which both currency and stored value are used to make payments. Stored value cards are provided by a competitive intermediary sector and are used in transactions for which the cost of stored value is less than the float cost associated with using currency. Conditions are identified under which the equilibrium allocation of the economy with stored value cards does or does not Pareto-dominate that of an otherwise identical economy without stored value cards. The critical condition is a boundary on the average cost of stored value: stored value is beneficial or harmful depending upon whether, other things equal, average cost is below or above a certain cut-off. If average cost is low, the reduction in the deadweight loss due to inflation will be large and the resource cost will be low. If average cost is high, the resource diversion will be large and there will be little effect on the burden of inflation.

The fact that costly private substitutes for government fiat money can reduce welfare was demonstrated by Schreft (1992a), and the model presented below is an extension of hers. This fact should not be surprising—as I argue below, we should expect the same result in any model with multiple means of

---

<sup>2</sup> See the appendix for documentation.

payments.<sup>3</sup> The interest foregone by holding currency is an opportunity cost to private agents, and they are willing to incur real resource costs to avoid it. The resource cost of a money substitute is a social cost, while the interest cost associated with currency is not. Thus the private incentive to provide a substitute for government currency is greater than the social benefit, a point stressed by Wallace (1986). He has argued that a positive nominal interest rate provides a similar incentive to issue private banknotes (Wallace 1983). While banknotes employ virtually the same technology as government currency, stored value employs a very different technology but serves the same role—both are private substitutes for government currency.

The best policy in the model is one in which the nominal interest rate is zero—the Friedman rule for the optimum quantity of money (Friedman 1969). For a given positive nominal interest rate, however, Schreft (1992a) has shown that quantitative restrictions on the use of credit as a means of payment can improve welfare. The same is true for stored value as well, since stored value is just another form of credit as a means of payment; if nominal interest rates are positive, then the right kind of quantitative constraints on stored value cards, if practical, can improve welfare by preventing the most wasteful uses. A non-interest-earning reserve requirement on stored value liabilities is inferior to quantitative constraints because it imposes an inframarginal tax on users of stored value.

No attention is paid here to consumer protection or to the safety and soundness of bank stored value activities (see Blinder [1995] and Laster and Wenninger [1995]). The analysis presumes that stored value systems provide relatively fraud- and counterfeit-proof instruments. Historical instances of private issue of small-denomination bearer liabilities, such as the early nineteenth century U.S. “free banking” era, have raised concerns about fraudulent note issue (see Friedman [1960]). Williamson (1992) shows that a government monopoly in issuing circulating media of exchange may be preferable to *laissez-faire* private note issue due to asymmetric information about bank portfolios. Others argue that a system of private banknote issue can function rather well: see Rolnick and Weber (1983, 1984). In any case, current communications technologies and regulatory and legal restraints are quite different from those of the early nineteenth century. Whether the concerns of Friedman and Williamson are relevant to stored value cards is beyond the scope of this article; the focus here is the implication of the seigniorage tax for the private incentives to provide currency substitutes.

---

<sup>3</sup> For models of multiple means of payment see Prescott (1987), Schreft (1992a, 1992b), Marquis and Reffett (1992, 1994), Ireland (1994), Dotsey and Ireland (1995), English (1996), and Lacker and Schreft (1996).

## 1. STORED VALUE CARDS

Stored value cards—sometimes called “smart cards”—contain an embedded microprocessor and function much like currency for a consumer. Value is loaded onto a card at a bank branch, an automated teller machine (ATM), or at home through a telephone or computer hookup with a bank. Customers pay for the value loaded onto the card either by withdrawing funds from a deposit account or by inserting currency into a machine. Customers spend value by sliding it through a merchant’s card reader, which reduces the card’s balance by the amount of the purchase and adds it to the balance on the merchant’s machine. Merchants redeem value at the end of the day through a clearing arrangement similar to those used for “off-line” credit card or ATM transactions. The merchant dials up the network and sends in the stored value, which is then credited to the merchant’s bank account. More elaborate systems allow consumers to transfer value from card to card.

The value on a stored value card is a privately issued bearer liability. It is different from a check because the merchant does not bear the risk of insufficient funds in the buyer’s account. It is different from a debit card in that the consumer hands over funds upon obtaining the stored value, while a debit card leaves the funds in the consumer’s account until the transaction is cleared. Thus a debit card is a device for authorizing deposit account transfers, while a stored value card records past transfers.

For consumers, stored value cards can be more convenient than currency in many settings; some consumers are likely to find cards physically easier to handle than coins and paper notes. The technology could conceivably allow consumers to load value onto their cards using a device attached to their home computer, saving the classic “shoe leather” cost of bank transactions. For merchants, stored value cards offer many of the advantages of credit card sales. The merchant saves the trouble of handling coins and notes (banks often charge fees for merchant withdrawals and deposits of currency) and avoids the risk of employee theft. Stored value improves on the mechanisms used for credit and debit cards, however, because the merchant’s device verifies the validity of the card, without costly and time-consuming on-line authorization. Thus stored value cards extend the electronic payments technology of credit card transactions to more time-sensitive settings where on-line authorization is prohibitive.

## 2. A MODEL OF CURRENCY AND STORED VALUE

This section describes a model in which stored value and currency both circulate in equilibrium. Monetary assets are useful in this model because agents are spatially separated and communication is costly. In the absence of stored value, agents use currency whenever shopping away from home, and the structure of

the model is reduced to a simple cash-in-advance framework. Stored value is a costly private substitute for currency, similar to costly trade credit in Schreft's (1992a, 1992b) models.

The model is a deterministic, discrete-time, infinite-horizon environment with a large number of locations and goods but no capital. At each location there are a large number of identical households endowed with time and a technology for producing a location-specific good. Each period has two stages. The first takes place before production and, in principle, allows any agent to trade any contingent claim with any other agent. During the second stage, production and exchange take place. One member of the household—the “shopper”—travels to other locations to acquire consumption goods. Simultaneously, the other member of the household—the “merchant”—produces location-specific goods and sells them to shoppers from other locations. Shoppers are unable to bring with them goods produced in their own location, so direct barter is infeasible.

The fundamental friction in this environment is that it is prohibitively costly for agents from two different locations to verify each other's identities. As a result, intertemporal exchange between agents from different locations is impossible, or, more precisely, not incentive compatible. Meetings between shoppers and merchants away from home thus effectively resemble the anonymous meetings of the Kiyotaki-Wright (1989) model. This provides a role for valued fiat currency. Because agents from the same location are known to each other, households can exchange arbitrary contingent claims with other households at the same location during the “securities market” in the first stage of each period. Households could travel to other locations during the securities market, but anonymity prevents meaningful exchange of intertemporal claims.

The stored value technology is a costly way of overcoming this friction. There are a large number of agents that are verifiably known to all—call them “issuers.” They are price-takers and thus earn a competitive rate of return. Like all other agents they can travel to any other location during the securities market, but since their identities are known, they are able to issue enforceable claims. The claim people want to buy is one they can use in exchange in the goods market. The difficulty facing such an arrangement, however, is authenticating the claim to the merchant—in other words, the difficulty of providing the shopper with a means of communicating the earlier surrender of value. Issuers possess the technology for creating message-storage devices—“stored value cards”—that shoppers can carry and machines that can read and write messages on these devices. Issuers offer to install machines with merchants. These machines can read, verify, and write messages on shoppers' cards and can record and store messages.

In principle the stored value technology described here could be configured to communicate any arbitrary messages. In this setting, however, a very simple message space will suffice. The shopper's device carries a measure of

monetary value, and the merchant's device deducts the purchase price from the number on the shopper's card and adds it to the measure of value stored on the merchant's device. During the next day's securities market, the issuer visits the merchant and verifies the amount stored on the reader. The issuer sells stored value to households in securities markets and then redeems stored value from merchants during the next day's securities markets. Messages in this case function much like the tokens in Townsend's (1986, 1987, 1989) models of limited communication. From this perspective, currency and stored value can be seen as alternative communications mechanisms.

I will adopt a very simple assumption concerning the cost of the stored value technology. I will assume that the card-reader devices that merchants use are costless to produce but require maintenance each period and that only issuers have the expertise to perform this maintenance. The amount of maintenance required depends on the location in which the device is installed and is proportional to the real value of the transactions that were recorded on the device. Some locations are more suited to stored value systems than others. This assumption will allow stored value and currency to coexist in equilibrium, with currency used at the locations that are less well-suited for stored value. The proportionality of costs to value processed might reflect security measures or losses due to fraud that rise in proportion to the value of transactions. I make no effort to model such phenomena explicitly but merely take the posited cost function as given. There are no other direct resource costs. In particular, stored value cards themselves are costless.<sup>4</sup>

The stored value cost function here is quite simple and in many respects somewhat unrealistic. Merchants' devices and the communications networks used to "clear" stored value are, arguably, capital goods and should be represented as investment expenditures rather than input costs. I am abstracting from capital inputs here, but this seems appropriate in a model with no capital goods to begin with.<sup>5</sup> Another feature of my cost function is that cost is proportional to the value of the transaction. In practice, the resource cost of electronic storage and transmission might not vary much with the numerical value of the message: transmitting "ten" should not be much cheaper than transmitting "ten thousand." Thus it seems plausible that a communications system, once built, would be

---

<sup>4</sup> Indirect evidence suggests that the resource costs of stored value systems could be substantial. A variety of sources indicate that bank operating expenses associated with credit cards amount to around 3 or 4 percent of the value of credit card charge volume. This does not include the direct expenses of merchants such as the costs of card readers. Stored value systems may avoid some expenses such as the costs associated with credit card billing and the cost of on-line communications. On the other hand, the cost of stored value cards themselves are greater than the cost of traditional "magnetic strip" cards.

<sup>5</sup> See Ireland (1994), Marquis and Reffett (1992, 1994), and English (1996) for models with private payment technologies requiring capital goods.

equally capable of carrying large and small value messages.<sup>6</sup> The cost function I adopt is the simplest one that is sufficient to demonstrate the claim that the introduction of stored value cards can reduce economic welfare. One critical feature is that the relative opportunity costs of stored value and currency vary across transactions so that both potentially circulate in equilibrium. A second critical feature is that there are constant returns to scale in providing stored value at any given location so that competition among providers is feasible. It should become clear as I proceed that the results are likely to carry over to settings with more elaborate cost functions.

The assumed technology also implies that the choice between currency and stored value takes a particularly simple form. The cost of using currency is the interest foregone while it is in use. The cost of stored value is simply the resource cost described above. By assuming that government currency is costless, I am abstracting from many of the factors mentioned in the previous section such as physical convenience, currency handling costs, and employee theft. Costless currency simplifies the presentation without loss of generality. In the appendix I describe a model in which there are private costs of handling currency and show that Proposition 1 below still holds. One could also modify the model to include government currency costs, as in Lacker (1993). The appendix also contains a model in which stored value substitutes for other more costly means of payment such as checks or credit cards; Proposition 1 holds in that model as well.

I can now begin describing the model more formally.<sup>7</sup>

### Households

Time is indexed by  $t \geq 0$ , and locations are indexed by  $z$  and  $h$ , where  $z, h \in [0, 1)$ . For a typical household at location  $h \in [0, 1)$ , consumption of good  $z$  at time  $t$  is given by  $c_t(h, z)$ , and labor effort is given by  $n_t(h)$ . Households are endowed with one unit of time that can be devoted to labor or leisure. The production technology requires one unit of labor to produce one unit of consumption good. Household preferences are

$$\sum_{t=0}^{\infty} \beta^t u(c_t(h), 1 - n_t(h)), \quad c_t(h) = \inf_z c_t(h, z), \quad (1)$$

---

<sup>6</sup> Ireland (1994) studies a similar model in which cost is independent of the value of the transaction. Also see Prescott (1987) and English (1996). For a partial equilibrium model see Whitesell (1992).

<sup>7</sup> The model is most closely patterned after the environment in Schreft (1992a) and Lacker and Schreft (1996). The main difference is that here the alternative payments medium is used at a subset of locations by all shoppers visiting that location, rather than at all locations but by only a subset of shoppers visiting that location. Also, I allow a general cost function while Schreft's cost function is, for convenience, linear in distance.

where  $u$  is strictly concave and twice differentiable. Household preferences are thus Leontieff across goods. This assumption implies that the composition of consumption is unaffected by the relative transaction costs at different locations, which considerably simplifies matters. In addition, it implies that transaction costs at a given location are passed on entirely to shoppers, since demand at a given location is inelastic.<sup>8</sup> Since all goods will bear a positive price in equilibrium, we can assume without loss of generality that  $c_t(h, z) = c_t(h)$  for all  $z$ .

In the securities market households acquire both currency and stored value. Since the units in which value is stored are arbitrary, there is no loss in generality in assuming that stored value is measured in units of currency. Thus one dollar buys one unit of stored value. Let  $c_t^m(h, z)$  and  $c_t^s(h, z)$  denote the consumption of good  $z$  at time  $t$  purchased with currency and stored value, respectively. Let  $m_t(h)$  and  $s_t(h)$  be the amount of currency and stored value acquired in the securities market at time  $t$ . Then the trading friction implies that

$$m_t(h) \geq \int_0^1 p_t c_t^m(h, z) dz \quad \text{and} \quad (2)$$

$$s_t(h) \geq \int_0^1 p_t^s(z) c_t^s(h, z) dz, \quad (3)$$

where  $p_t$  is the price of goods for currency and  $p_t^s(z)$  is the price of goods for stored value at location  $z$ .<sup>9</sup>

Household  $h$  sells  $y_t^m(h)$  units of output for currency and  $y_t^s(h)$  units of output for stored value. In addition, they sell  $y_t^i(h)$  units of output to issuers. Since issuers are well known, merchants are willing to sell to them on credit and accept payment in next period's securities market. Feasibility requires

$$y_t^m(h) + y_t^s(h) + y_t^i(h) \leq n_t(h). \quad (4)$$

At the end of the period, the household has  $p_t^s(h)y_t^s(h)$  units of stored value on their card reader to be redeemed at  $t + 1$ . Issuers pay interest at the nominal rate  $i_t$ , the market rate on one-period bonds, but deduct a proportional charge at rate  $r_t(h)$  from the proceeds to cover their costs. Thus the household receives

---

<sup>8</sup> Allowing substitution between goods would imply that the composition of consumption varies with changes in relative transaction costs. Relative prices net of transaction costs would then vary across locations, destroying the symmetry in households' consumption and leisure choices. See Ireland (1994) and Dotsey and Ireland (1995) for models which relax the Leontieff assumption.

<sup>9</sup> In principle the price of goods for currency could vary across locations as well, but symmetry will ensure equality across locations. This is confirmed below: see (12). Note that shoppers do not receive explicit interest on stored value. Note also that I allow merchants to charge a different price for different payments instruments.



$[1 - r_t(h)](1 + i_t)p_t^s(h)y_t^s(h)$  units of currency at  $t + 1$  for the stored value they have accepted.<sup>10</sup>

Households bring the following to the securities market: currency from the previous period's sales, stored value to be redeemed, maturing bonds, and any currency that might be left over from shopping in the previous period. Letting  $b_t$  be bond purchases at  $t$ , and  $\tau_t$  be lump-sum taxes at  $t$ , households face the following budget constraint at time  $t + 1$ :

$$\begin{aligned} m_{t+1}(h) + s_{t+1}(h) + b_{t+1} + \tau_{t+1} \leq & m_t(h) - \int_0^1 p_t c_t^m(h, z) dz + s_t(h) \\ & - \int_0^1 p_t^s(z) c_t^s(h, z) dz + (1 + i_t)b_t + p_t[y_t^m(h) + y_t^i(h)] \\ & + [1 - r_t(h)](1 + i_t)p_t^s(h)y_t^s(h). \end{aligned} \quad (5)$$

Households maximize (1) subject to (2) through (5) and the relevant nonnegativity constraints, taking prices and interest rates as given.

### Issuers

There are a large number of issuers at location zero, distinct from the households described above. Their preferences depend on their consumption  $[c_t^i(z)]$  and leisure  $(1 - n_t^i)$  according to

$$\sum_{t=0}^{\infty} \beta^t (c_t^i - n_t^i), \quad c_t^i = \inf_z c_t^i(z). \quad (6)$$

Issuers sell  $s_t(z)$  units of stored value per capita in securities market  $z$  at time  $t$  in exchange for  $s_t(z)$  units of currency and redeem  $s_t'(z)$  units of stored value per capita from merchants at market  $z$  at time  $t + 1$  in exchange for  $[1 - r_t(z)](1 + i_t)s_t'(z)$  units of currency. All of the stored value they issue will be spent by households and then redeemed from card readers in equilibrium, so issuers face the constraint

$$\int_0^1 s_t(z) dz = \int_0^1 s_t'(z) dz. \quad (7)$$

Maintenance of the devices on which the stored value at location  $z$  is redeemed requires  $\gamma(z)s_t'(z)/p_t^s(z)$  units of labor effort, where  $\gamma(z)$  is a continuous, strictly increasing function with  $\gamma(0) = 0$ . Total maintenance effort is therefore

$$n_t^i = \int_0^1 \gamma(z)s_t'(z)/p_t^s(z) dz. \quad (8)$$

<sup>10</sup> Note that the payment could be thought of as redemption at par plus a premium  $[1 - r_t(h)](1 + i_t) - 1$ . The form of merchants' payments to issuers depends on issuers' cost functions. The payment is proportional to the value redeemed because the issuers' costs are proportional to value redeemed. If costs were independent of value redeemed, the equilibrium payment would also be independent of value redeemed.

The alternative for issuers who are not active is to consume nothing [ $c_t^i(z) = n_t^i = 0$ ], which can be interpreted as the proceeds of some alternative autarchic activity.

The family of an issuer consists of a worker and a shopper. The worker manages the stored value business, while the shopper travels around during the goods market period purchasing consumption. Since issuers are known to all, the shopper can buy on credit, paying  $p_t c_t^i(z)$  in the location- $z$  securities market at  $t + 1$  for goods purchased there at  $t$ . Excess funds are invested in bond holdings  $b_t^i$ . An issuer's securities market budget constraint is

$$p_t c_t^i + b_{t+1}^i \leq \int_0^1 s_{t+1}(z) dz + (1 + i_t) b_t^i - \int_0^1 (1 + i_t) [1 - r_t(z)] s_t^i(z) dz. \quad (9)$$

Active issuers maximize (6), subject to (7) through (9) and the relevant non-negativity constraints, by choosing consumption, bonds, labor effort, and the amount of stored value to issue and redeem at each location. Because there are a large number of issuers, competition between them will drive the utility of active issuers down to the reservation utility associated with inactivity. Issuers initially have no assets.

### Government

The government issues fiat money  $M_t$  and one-period bonds  $B_t$ , collects lump-sum taxes  $T_t$ , and satisfies

$$M_{t+1} + B_{t+1} = M_t + (1 + i_t) B_t - T_{t+1} \quad (10)$$

for all  $t$ . The government sets a constant money growth rate  $\pi = M_{t+1}/M_t - 1$ , where  $\pi \geq \beta - 1$ .

### Equilibrium

A symmetric monetary equilibrium consists of sequences of prices, quantities and initial conditions  $M_{-1}$  and  $(1 + i_{-1})B_{-1}$ , such that households and issuers optimize, the lifetime utility of active issuers is equal to the lifetime utility of inactive issuers starting at any date, the government budget constraint (10) holds, and market clearing conditions hold for  $M_t, B_t, y_t^m(h), y_t^s(h)$ , and  $y_t^i(h)$  for all  $t$  and  $z$ . I restrict attention to stationary equilibria, in which real magnitudes are constant over time. Where possible, time subscripts will be dropped from variables that are constant over time; variables refer to date  $t$  quantities unless otherwise noted.

The first-order necessary conditions for the issuer's maximization problem imply

$$r(z)(1 + i) = \gamma(z) p_t / p_t^s(z). \quad (11)$$

The left side of (11) is the nominal net return from issuing one dollar's worth of stored value at  $t$  to be redeemed at location  $z$  at  $t + 1$ , with the proceeds invested in a bond maturing at  $t + 1$ . Interest on the bond is paid over to the merchant, with a portion  $r(z)$  of the payment deducted as a fee. The right side of (11) is the nominal cost of enough consumption goods to compensate the issuer for the disutility of maintaining the stored value device at location  $z$ . Thus condition (11) states that for stored value issuers marginal net revenue equals marginal cost at each location.

Merchants consider whether to sell output for currency or for stored value. The first-order necessary conditions for the household's maximization problem imply that if merchants are indifferent between accepting currency and stored value, then

$$p_t = [1 - r(z)](1 + i)p_t^s(z). \quad (12)$$

As a result, the last two terms in the household's budget constraint (5) simplify to  $p_t n(h)$ . Households at all locations face identical terms of trade between consumption and leisure despite the difference in transaction costs across locations. Consumption and labor supply are therefore identical across locations and the notation for  $h$  can be suppressed.

When shoppers consider whether to use currency or stored value to purchase consumption at location  $z$ , they compare the unit cost of the former,  $p_t$ , to the unit cost of the latter,  $p_t^s(z)$ . Using (11) and (12), shoppers use stored value if

$$p_t > p_t^s(z) = p_t[1 + \gamma(z)]/(1 + i). \quad (13)$$

Thus stored value is used where  $\gamma(z) < i$ . Because  $\gamma$  is strictly increasing, the boundary between the stored value and the currency locations can be written as a function  $\zeta(i) \equiv \gamma^{-1}(i)$ . Stored value will coexist with currency as long as  $i$  is less than  $\gamma(1)$ , the cost of stored value at the highest cost location. If  $i \geq \gamma(1)$ , then stored value drives out currency; in this case  $\zeta(i)$  is one.

The total resource cost of stored value for a given nominal rate is

$$\int_0^{\zeta(i)} \gamma(z) dz \equiv \Gamma(i). \quad (14)$$

Steady-state equilibrium values of  $c$  and  $n$  can be found as the solutions to the first-order condition

$$u_1(c, 1 - n)/u_2(c, 1 - n) = 1 + [1 - \zeta(i)]i + \Gamma(i), \quad (15)$$

along with the feasibility condition

$$n = c[1 + \Gamma(i)]. \quad (16)$$

For a given nonnegative nominal interest rate, consumption and employment are determined by (15) and (16).<sup>11</sup> Assuming that neither leisure nor consumption are inferior goods, then  $v(c, 1 - n) \equiv u_1(c, 1 - n)/u_2(c, 1 - n)$  is decreasing in  $c$  and increasing in  $1 - n$ . If in addition we assume that  $v(c, 1 - n)$  goes to infinity as  $c$  goes to zero and zero as  $1 - n$  goes to zero, then we are guaranteed an interior solution; the proof appears in the appendix. The real interest rate is  $\beta^{-1} - 1$  in all equilibria, and the inflation rate is  $\beta(1 + i) - 1$ .

Without stored value cards the economy has the same basic structure as a standard cash-in-advance model (Lucas and Stokey 1983) and can be obtained as a special case in which  $\zeta(i)$  (and thus  $\Gamma(i)$ ) equals zero.

### 3. THE WELFARE ECONOMICS OF STORED VALUE

An optimal steady-state allocation is defined by the property that no other feasible steady-state allocation makes at least one type of household better off without making some other type of household worse off. Two features of the model make optimality relatively easy to assess. Even though at some locations goods are sold for currency and at other locations goods are sold for stored value, households at all locations face identical terms of trade between consumption and leisure. As a result, all households at all locations will have the same lifetime utility in any given equilibrium. We can therefore focus our attention on the well-being of a representative household at a representative location. Second, because the lifetime utility of any given issuer is zero in all equilibria, we can effectively ignore the welfare of issuers when comparing equilibrium allocations. This just reflects the fact that issuers receive a competitive rate of return and are indifferent as to how they obtain it; constant returns to scale in providing stored value at any location implies that issuers earn no rents. Given these two features, we can compare alternative allocations by considering their effect on the lifetime utility of a representative household.

For the version of this economy without stored value, the welfare economics are well known. Households equate the marginal rate of substitution between consumption and leisure to  $1 + i$  rather than 1, the marginal rate of transformation, because consumption is provided for out of currency accumulated by working in the previous period, and currency holdings are implicitly taxed at rate  $i$ . Optimality requires that the marginal rate of substitution equal the marginal rate of transformation, which only holds if the nominal interest rate is zero. A positive nominal interest rate distorts household decisions,

---

<sup>11</sup> The reduced form structure in (15) and (16) is identical to Schreft (1992a) and Lacker and Schreft (1996), although the models are somewhat different. In Schreft's model households use credit when close to their own home location and currency when farther away; thus at every location shoppers from nearby use credit and shoppers from a distance use cash. In contrast, at some locations only stored value is used and at other locations only currency is used in my model. Also, in Schreft's model credit costs are linear in distance.

inducing substitution away from monetary activity (consumption) toward non-monetary activity (leisure). The resulting welfare reduction is the deadweight loss from inflation in this model. Intuitively, inflation reduces the rate of return on currency, which causes consumers to economize on the use of currency. In a cash-in-advance economy they can do this only by consuming less of the goods whose purchase requires currency. The optimal monetary policy is to deflate at the rate of time preference,  $\pi = \beta - 1$ , so that the nominal interest rate is zero and the distortion in (15) is completely eliminated (Friedman 1969). Note that in the absence of stored value, inflation has no effect on the feasibility frontier (16).

I will compare two steady-state equilibria with identical inflation rates, one with and one without stored value. Since the real rate is the same in all equilibria, the nominal rate is constant across equilibria as well. Stored value has two effects on a typical household's utility. The first is to alter the marginal rate of substitution between monetary and nonmonetary activities. The transaction cost associated with purchases using stored value at a given location  $z < \zeta(i)$  is  $\gamma(z)$ , which is less than  $i$ , the private opportunity cost associated with using currency: see Figure 1. Thus stored value reduces the average transaction cost associated with consumption goods. This can be seen from (15), noting that the average cost of stored value,  $\Gamma(i)/\zeta(i)$ , is less than  $i$ . Stored value reduces the right side of (15) by the amount  $\zeta(i)i - \Gamma(i)$ , shown as the region *A* in Figure 1. Therefore, stored value cards reduce the distortion caused by inflation. By itself, this increases welfare. Note that the lower the total cost of stored value  $\Gamma(i)$ , holding constant  $i$  and  $\zeta(i)$ , the larger the welfare gain from stored value.

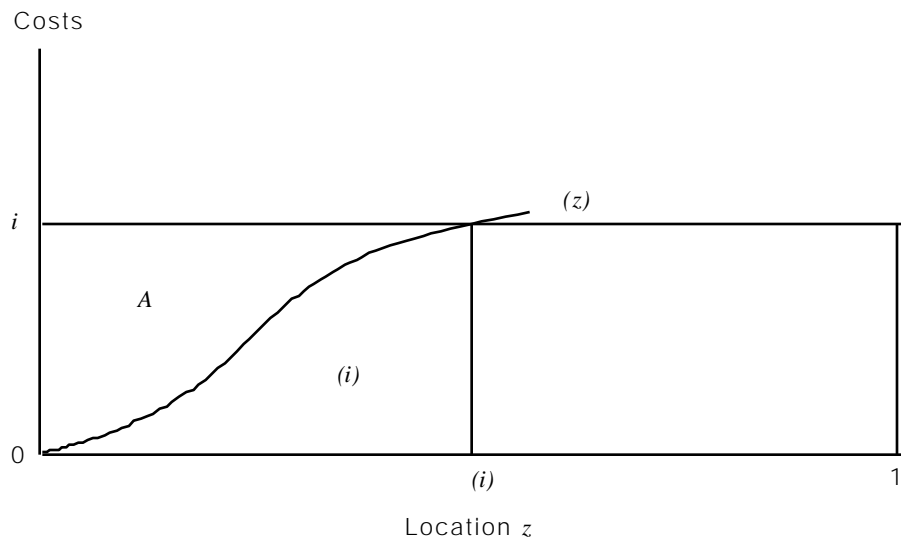
The second effect is through the feasibility constraint. Stored value cards involve real resource costs. Maintenance of the technology requires issuers' labor time, and consumption at every location must be diverted to compensate issuers for their effort. Currency requires no direct resource costs in this model.<sup>12</sup> The introduction of stored value shifts the consumption-leisure feasibility frontier (16) inward, since resources must be diverted to cover the real costs of stored value. The area under  $\gamma(z)$  from zero to  $\zeta(i)$  in Figure 1 is equal to  $\Gamma(i)$ , the real resource costs devoted to stored value activities. In contrast, the opportunity cost associated with currency (the area under  $i$ ) is merely a transfer payment. By itself, the resource cost of stored value reduces welfare; a virtually costless government money is replaced by a costly private money. Note that the larger the total cost of stored value (holding constant  $i$  and  $\zeta(i)$ ), the larger the reduction in welfare.

The net effect of stored value cards on economic welfare is indeterminate and depends on the structure of stored value costs across locations. Since we

---

<sup>12</sup> See the appendix for a model with positive private costs of using currency and Lacker (1993) for a similar model with positive government currency costs.

**Figure 1 Opportunity Costs for Currency and Stored Value across Locations**



are comparing two equilibria with the same nominal interest rate, we know that the marginal location has a cost of  $i$ . But conditions (15) and (16) tell us that the effect depends on the shape of the cost function. The benefit of stored value in (15) varies positively with the area  $A$  in Figure 1. The detrimental effect of stored value in (16) varies positively with  $\Gamma(i)$ . Both effects depend solely on  $\Gamma(i)$  for any given  $i$  and  $\zeta(i)$ . If costs are low across most locations but then rise sharply—for example, if  $\gamma$  is quite convex—then  $\Gamma(i)$  will be relatively small. In this case the negative effect through the feasibility condition will be small, and the positive effect through the marginal rate of substitution,  $\zeta(i)i - \Gamma(i)$ , will be large. If instead costs are large at most locations—for example, if  $\gamma$  is quite concave—then  $\Gamma(i)$  is close to  $\zeta(i)i$ , the resource costs will be large, and the gain from reducing the marginal rate of substitution will be small. Thus the greater the convexity of costs across locations, the more likely it is that stored value cards improve economic welfare. This intuition is formalized in the following proposition. (The proof appears in the appendix.)

**Proposition 1:** Fix  $i$ . Compare an economy with no stored value to an arbitrary stored value economy with a given ratio of stored value to currency,  $\zeta(i)/[1 - \zeta(i)]$ . There is a cutoff  $\Gamma^*$  [which depends on  $i$  and  $\zeta(i)$ ] such that if  $\Gamma(i) > \Gamma^*$ , then welfare is lower in the stored value economy, and if  $\Gamma(i) < \Gamma^*$ , then welfare is higher in the stored value economy.

The principle described in Proposition 1 appears to be quite general. In any model in which there is a deadweight loss due to the inflation tax, a private substitute for currency will reduce the base on which the tax is levied. The cost of the substitute must be less than the tax it evades—otherwise it would not be introduced. With the tax rate (the nominal interest rate) held constant, the incidence of the tax is lower and so the deadweight loss associated with that distortion will fall. Thus in any model we should expect that private substitutes for fiat money help reduce the burden of a given inflation rate.

The negative welfare effect of stored value cards would seem to generalize as well. Absent market imperfection, participants will adopt stored value if their collective private benefits exceed their collective private costs. But their net benefits differ from social net benefits in two ways; the capture of seigniorage is not a social benefit, and they do not bear the governmental cost of currency provision. The gap between the nominal interest rate and the government's per-dollar cost of providing currency thus represents the *excess* incentive to implement currency substitutes.

More realistic or elaborate models would also display the principle described in Proposition 1. For example, if stored value costs do not depend directly on the value of messages, then the fee an issuer collects from a merchant would be independent of the merchant's sales.<sup>13</sup> The relevant cost comparison is then between the seigniorage tax, which is proportional to production, and the fixed fee, which is not. In this case the set of locations using stored value would vary with equilibrium consumption, instead of being independent of consumption as in the model above. Nevertheless, stored value would reduce transaction costs where it is used, and the benefit of a reduced inflation tax burden would have to be weighed against the added resource costs.

In the absence of stored value, inflation is costly in the model because it distorts the choice between monetary and nonmonetary activities. Some economists have suggested that an important cost of inflation is that it encourages costly private credit arrangements as substitutes for government money (see Ireland [1994], Dotsey and Ireland [1995], Lacker and Schreft [1996], and Aiyagari, Braun, and Eckstein [1995]). Stored value could reduce this cost of inflation by displacing even more costly means of payment such as checks or credit cards. The social benefit of stored value would then also include the reduction in payments cost for some transactions. This benefit would be larger, the smaller the cost of stored value. But again, the benefit of stored value would have to be weighed against the resource cost of substituting stored value for virtually costless currency. As long as stored value in part substitutes for currency, Proposition 1 again emerges; the lower the average cost of stored value cards, the greater the gain from displacing more costly means of payments, and

---

<sup>13</sup> The payments technologies in Ireland (1994) and English (1996) have this property.

the smaller the resources diverted to stored value systems. (A straightforward extension of the model demonstrating this result is described in the appendix.)

As I mentioned earlier, stored value seems likely to offer consumers and merchants greater physical convenience or some other advantages over currency that are not captured in the model presented above. Such advantages would not alter the basic feature of Proposition 1, however. If merchants find stored value less costly to handle than currency, their savings will presumably be reflected in their willingness to pay stored value issuers; the social benefit of stored value to merchants will be reflected in issuers' revenues. Similarly, if consumers find stored value more convenient than currency, they should be willing to pay, either directly or indirectly, and the benefit of stored value to consumers will be reflected in issuers' revenues. The greater convenience of stored value cards will provide issuers with added incentive to provide stored value, but the nominal interest rate (minus the government currency cost) would *still* constitute a source of private return to issuing stored value that is not matched by any social benefit of replacing currency. To demonstrate this point, the appendix describes a simple extension of the model in which there are private costs to handling currency and shows that Proposition 1 again holds true.

What if consumers earn interest on their stored value cards, a possibility that appears to be technologically feasible? Could this upset the conclusions of Proposition 1? In the model, merchants earn interest on stored value balances, although they pay some of this interest back to issuers in the form of redemption fees. Stored value yields no explicit interest for shoppers. An equivalent scheme would be for shoppers to earn interest on stored value but face higher prices at locations that accept stored value. The interest earnings would more than compensate shoppers for the higher price at those locations. Stored value would again be used at locations at which the resource cost does not exceed the opportunity cost of using currency.

What if stored value completely displaces currency? In this case the meaning of the nominal interest rate in the model becomes somewhat ambiguous; because currency does not circulate, it might not serve as the unit of account. Nonetheless, the difference between the real return on bonds and the real return on stored value liabilities will not exceed the marginal cost of providing stored value.<sup>14</sup> Unless this difference is less than the government's cost of providing currency, the principle underlying Proposition 1 still applies.

Bryant and Wallace (1984) have argued that different rates of return on different government liabilities can be justified as an optimal tax. If all sources

---

<sup>14</sup> This could happen in one of two ways. Currency could remain the unit of account—a “ghost” money. Issuers would pay interest to consumers on stored value, and issuers' net interest margin would not exceed the marginal cost of stored value. Alternatively, stored value could become the unit of account, in which case the nominal interest rate would fall to the marginal cost of stored value.



of government revenue require distortionary taxation, then it may be beneficial to raise some revenues from the taxation of currency holders. This consideration is not captured in the model described above. The seigniorage tax merely finances interest payments on government bonds. The reduction in seigniorage revenues was offset by a reduction in outstanding government debt or an increase in lump-sum taxes, keeping the nominal rate constant. If instead the loss of seigniorage had to be recovered by raising other distortionary taxes, it would strengthen the case against stored value. The additional deadweight burden of the compensating tax increases would have to be added to the resource cost of stored value. Similarly, nonzero government expenditures financed in part through seigniorage would not change the basic features of Proposition 1.

#### 4. POLICY

In the presence of one distortionary tax a second distortion can sometimes improve economic welfare. A positive nominal interest rate is a distortionary tax on holders of government currency. In the laissez-faire regime with stored value, welfare can be lower because of the costs of stored value, which suggests that constraints on the issue of stored value cards might be welfare-enhancing. This turns out to be true. Restrictions on the issue of stored value can improve welfare, in this second-best situation, by reducing the costly displacement of government currency.<sup>15</sup>

Consider first a simple quantitative restriction on the use of stored value. Imagine a legal restriction that limits the quantity of stored value used by households, or, equivalently, that limits the locations at which stored value is accepted. Households can only make a fraction  $\eta$  of their purchases using stored value, where the government sets  $\eta$  between 0 and  $\zeta(i)$ . Households will continue to use stored value where it is most advantageous to do so—at locations 0 through  $\eta$ . At locations  $\eta$  through 1, households use currency. Equilibrium is still characterized as the solution to (15) and (16), except that  $\zeta(i)$  is replaced by  $\eta$ . For a given nominal interest rate, consumption and employment are determined as the solutions to

$$u_1(c, 1 - n)/u_2(c, 1 - n) = 1 + (1 - \eta)i + \int_0^{\eta} \gamma(z)dz \quad \text{and} \quad (17)$$

$$n = c[1 + \int_0^{\eta} \gamma(z)dz]. \quad (18)$$

Define  $c(\eta)$  and  $n(\eta)$  as the solutions to (17) and (18) for  $\eta \in [0, \zeta(i)]$ , and  $v(\eta) \equiv u(c(\eta), 1 - n(\eta))$ . The function  $v(\eta)$  is the equilibrium utility of a

---

<sup>15</sup> This depends, of course, on finding a practical way to restrict the quantity of stored value issued. I do not address this issue here.

representative household under the constraint that no more than a fraction  $\eta$  of purchases can be made using stored value.

**Proposition 2:**  $v'[\zeta(i)] < 0$ ; therefore there is a binding restriction  $\eta < \zeta(i)$  on stored value under which steady-state utility is higher than under the *laissez-faire* regime.

Reducing  $\eta$  marginally below  $\zeta(i)$  has two effects. The direct effect via the resource constraint (18) is to eliminate the most costly uses of stored value, allowing greater consumption at each level of employment. The increase in utility at  $\eta = \zeta(i)$  is proportional to  $\gamma(i)$ . The second effect via the marginal rate of substitution is to substitute currency (transaction cost  $i$ ) for stored value (transaction cost  $\gamma(\eta)$ ). The fall in utility is proportional to  $i - \gamma(\eta)$ , which vanishes at  $\eta = \zeta(i) = \gamma^{-1}(i)$ , since at the margin stored value and currency bear the same transaction cost. The first-order resource savings dominates the negligible increase in the burden of inflation. The net effect of decreasing  $\eta$  is positive. Therefore there must be a value  $\eta < \zeta(i)$  that results in higher steady-state utility than the *laissez-faire* regime. Note that Proposition 2 holds whether or not the stored value equilibrium is worse than the no-stored-value equilibrium; even if stored value is welfare-enhancing, quantitative constraints would still be worthwhile.<sup>16</sup>

An alternative method of restraining stored value is to impose a reserve requirement. Issuers are required to hold currency equal to a fraction  $\delta$  of their outstanding stored value liabilities. Issuers earn  $(1 - \delta)i$  rather than  $i$  on their assets, and the cost of foregone earnings,  $\delta i$ , is passed on to merchants and ultimately to consumers. Stored value is used at fewer locations for any given interest rate—only where  $\gamma(i) < (1 - \delta)i$ . Raising the reserve requirement from zero reduces the amount of resources diverted to stored value. By itself this increases utility by easing the feasibility frontier in (18). The first-order condition (15) becomes

$$u_1(c, 1 - n)/u_2(c, 1 - n) = 1 + [1 - \zeta(i, \delta)]i + \int_0^{\zeta(i, \delta)} (1 - \delta)^{-1} \gamma(z) dz, \quad (19)$$

where  $\zeta(i, \delta) \equiv \gamma^{-1}[(1 - \delta)i]$ . Increasing the reserve requirement now has two effects on the marginal rate of substitution. First, raising  $\delta$  expands the set of

<sup>16</sup> Proposition 2 parallels Proposition 2 in Schreft (1992a). In Schreft's model the government issues no bonds and government expenditure depends on the seigniorage collected. Lowering the constraint holding the inflation rate constant increases the demand for money and thus government expenditures. Schreft's proposition requires the condition that government expenditures rise by less than the resource cost of payments services falls when the constraint is tightened. This condition is unnecessary if government spending is held constant and instead the bond supply or lump-sum taxes vary across equilibria. There are no government expenditures in my model. Note that Proposition 2 depends crucially on the continuity of stored value costs across locations. If there were a discrete jump in the function  $\gamma$  at  $\zeta(i)$ , then Proposition 2 might fail to hold.

locations at which currency is used instead of lower-cost stored value. This effect operates through  $\zeta(i, \delta)$  in (19). Second, raising  $\delta$  increases the transaction cost at all locations at which stored value is used. This effect increases the integrand in the last term in (19) and does not vanish at  $\delta = 0$ . The first effect is identical to the effect of decreasing the quantitative constraint  $\eta$  in (17). The quantitative constraint does not involve the second effect in (19), since it leaves inframarginal stored value users unaffected. In contrast, the reserve requirement imposes a tax on all stored value users. Therefore, a quantitative constraint is superior to a reserve requirement in this environment. A reserve requirement improves welfare only if the second effect in (19) does not dominate.<sup>17</sup>

## 5. CONCLUDING REMARKS

When the nominal interest rate is positive, there is an incentive to develop substitutes for government currency. This incentive is likely to exceed the social benefit of such substitutes because the private opportunity cost of holding currency is larger than the social cost of providing currency. Although stored value can lower the opportunity cost of payments media for inframarginal consumers, the real resources diverted to stored value are wasteful from society's point of view. For a given nominal interest rate, stored value cards are good or bad for welfare depending upon whether the average cost of stored value is below or above a certain cutoff. Quantitative restrictions on stored value can be socially beneficial, in this second-best situation, because they reduce the amount of resources absorbed by the most costly stored value applications. I do not claim to show that such restrictions can be easily implemented—only that if such restrictions were practical, they would enhance welfare.

Wallace (1983) pointed out that the U.S. government has effectively prohibited the private issue of paper small-denomination bearer notes such as bank notes. In the absence of such a ban, he argued, private intermediaries could issue perfect substitutes for government currency backed by default-free securities such as U.S. Treasury bills. In this case one of two things could occur. Either the nominal rate of return would not exceed the marginal cost of such intermediation, which he reckoned at close to zero, or government currency would cease to circulate. Stored value cards are just another way of issuing small-denomination bearer liabilities. As a corollary to Wallace's thesis, then, we should expect one of two things to happen. Either the nominal interest rate will not exceed the marginal cost of an additional dollar's worth of stored value, or government currency will cease to circulate. All I have added to Wallace's

---

<sup>17</sup> A reserve requirement equal to  $1 - \gamma_g/i$  could be imposed (where  $\gamma_g$  is the government currency cost) as an experiment to determine whether the resource costs of stored value exceed the direct benefits, i.e., whether  $\gamma(z) > \gamma_g$ . This experiment would not answer the question posed by Proposition 1, however.

argument is the observation that since stored value employs a technology that is different from, and potentially far more costly than, the government currency it would replace, it is possible that either outcome could reduce economic welfare.

The restrictions that have prevented private paper substitutes for currency were in place since at least the end of the Civil War. As Wallace (1986) notes, "(i)f there is a rationale for that policy . . . then it would seem that it would apply to other payments instruments that potentially substitute for the monetary base." These longstanding restrictions on paper note issue evidently have been repealed.<sup>18</sup> Current policy thus avoids the inconsistency of allowing electronic substitutes for government currency while preventing paper substitutes.

---

---

## **APPENDIX**

### **Government Currency Costs**

The Annual Report of the Director of the Mint reports the coinage cost per \$1,000 face value for every denomination of coin, along with the number manufactured. For 1993 (the latest year available) this yields a coin manufacturing cost of \$166.2 million (31.3 percent of face value). For coin operating cost the 1994 PACS Expense Report lists total cost of coin service of \$14.7 million (Board of Governors 1994b). Total government cost of coin is the sum of manufacturing and operating costs, or \$180.9 million. U.S. Treasury Department Bulletin reports coin in circulation on December 31, 1993, as \$20.804 billion. For coin, therefore, the cost per dollar outstanding is \$0.008695.

For currency, governmental cost for 1993 is the sum of Federal Reserve Bank operating expenses of \$123.7 million (Board of Governors 1994b), and the Reserve Bank assessment for U.S. Treasury currency expenses of \$355.9 million (Board of Governors 1994a). Total cost for currency is thus \$479.6 million, or \$0.001394 per dollar outstanding, based on \$343.925 billion in currency outstanding at the end of 1993 (Board of Governors 1994a).<sup>19</sup>

Combining currency and coin costs yields a total of \$660.5 million for 1993. The total value of currency and coin in circulation was \$36.729 billion. The total government cost of coin and currency per dollar outstanding is therefore \$0.001798.

---

<sup>18</sup> Title VI of the Community Development Banking Act, P.L. 103-325 (1994), repealed all restrictions on note issue by national banks except the 1/2 percent semi-annual tax on outstanding notes. Section 1904(a) of the Tax Reform Act of 1976 repealed the 10 percent tax on note issue by corporations other than national banks. De facto restrictions by bank regulators may still prevent private note issue.

<sup>19</sup> For more on the government's cost of currency, see Lacker (1993).

### Proofs

**Existence:** Define  $c(y)$  and  $n(y)$  as the joint solutions to  $u_1(c, 1-n) = qu_2(c, 1-n)$  and  $qc + (1-n) = y$ . Here  $q$  is the marginal rate of substitution between consumption and leisure, the right-hand side of (15). The assumptions on preferences imply that  $c(y)$  and  $1-n(y)$  are unique, strictly positive for  $y > 0$ , continuous, and monotone increasing. Assume  $r > 0$ , where  $r$  is the marginal rate of transformation between consumption and leisure, the bracketed term in (16). Then  $rc(y) + [1-n(y)]$  is strictly increasing in  $y$ , there exists a unique  $y$  such that  $rc(y) + [1-n(y)] = 1$ , and thus  $n(y) = rc(y)$ .

**Proposition 1:** Define  $c(q, r)$  and  $n(q, r)$  as the unique solutions to

$$\begin{aligned} u_1(c, 1-n) &= qu_2(c, 1-n) \\ n &= rc, \end{aligned}$$

where attention is restricted to  $r \geq 1$  and  $q \geq r$ . Define  $V(q, r) = u(c(q, r), 1-n(q, r))$ . It is easy to show that since neither leisure nor consumption is an inferior good,  $V$  is strictly decreasing in  $q$  and  $r$ .

The first-best allocation has a nominal interest rate of zero, so  $q = r = 1$ . Economies with positive nominal rates but no stored value have  $q = 1+i > 1$  and  $r = 1$ . For given  $i$  and  $\zeta(i)$ ,  $q$  and  $r$  vary positively with the aggregate  $\Gamma(i)$ . This amounts to varying average cost holding marginal cost constant at location  $z = \zeta(i)$ . Note that  $\Gamma(i)$  can lie anywhere in the interval  $(0, \zeta(i)i)$ . Define  $w(\Gamma) = V(1+(1-\zeta(i))i+\Gamma, 1+\Gamma)$ . Then  $w(0) = V(1+(1-\zeta)i, 1) > V(1+i, 1)$ , and  $w(\zeta(i)i) = V(1+i, 1+\zeta(i)i) < V(1+i, 1)$ . Since  $w(\Gamma)$  is continuous and strictly decreasing in  $\Gamma$ , it follows immediately that there exists a unique  $\Gamma^*$  for which  $w(\Gamma^*) = V(1+i, 1)$ .

**Proposition 2:** Define  $q(\eta)$  as the right side of (17), and  $r(\eta)$  as the bracketed term in (18). With  $v(\eta) \equiv V(q(\eta), r(\eta))$ , we have  $\lim_{\eta \rightarrow \zeta(i)} v'(\eta) = \lim_{\eta \rightarrow \zeta(i)} [V_q q' + V_r r'] = V_q [\gamma(\zeta(i)) - i] + V_r \gamma(\zeta(i)) = V_r \gamma(\zeta(i)) < 0$ , since  $\gamma(\zeta(i)) = i$  and  $V_r < 0$ .

### A Model in Which Stored Value Substitutes for Other Means of Payment

In this section I describe a simple extension of the model in which there are three means of payment: government currency, stored value, and another costly private means of payment. The latter can be thought of as checks or credit cards and is supplied by an industry with the same general properties as the stored value sector. Locations will now be indexed by  $z_1$  and  $z_2$ . Stored value costs depend only on  $z_1$  according to  $\gamma(z_1)$ . The cost of checks depends only on  $z_2$  according to a continuous and monotone increasing function  $\chi(z_2)$ . To simplify the exposition, I will abstract from the effect of inflation on labor supply and assume that labor is supplied inelastically:  $n \equiv 1$  and  $u(c, 1-n) \equiv u(c)$ .

Inflation is inefficient in this version of the model solely because it diverts resources to the production of money substitutes. Consumption equals output (which is fixed and equal to 1) minus the resource costs of checks and stored value. In the absence of stored value, shoppers use currency at locations where  $\chi(z_2) > i$ , and use checks where  $\chi(z_2) < i$ . Without stored value, then, the cost of inflation is

$$\int_0^1 \int_0^{\chi^{-1}(i)} \chi(z_2) dz_2 dz_1.$$

Shoppers will use stored value at locations where  $\gamma(z_1) < \min[\chi(z_2), i]$ . With stored value, the consumption diverted to alternative monies is

$$\int_{S(i)} \gamma(z_1) dz_2 dz_1 + \int_{C(i)} \chi(z_2) dz_2 dz_1,$$

where  $S(i) \equiv \{(z_1, z_2) \text{ s.t. } \gamma(z_1) < \text{MIN}[i, \chi(z_2)]\}$

and  $C(i) \equiv \{(z_1, z_2) \text{ s.t. } \chi(z_2) < \text{MIN}[i, \gamma(z_1)]\}$ .

As in the model in the text, stored value wastes resources: at locations described by  $z_1 < \gamma^{-1}(i)$  and  $z_2 > \chi^{-1}(i)$ , stored value costs are incurred where formerly (costless) currency was in use. At these locations, the greater the average cost of stored value the greater the resource diversion. However, at locations described by  $z_2 < \chi^{-1}(i)$  and  $\gamma(z_1) < \chi(z_2)$ , stored value substitutes for more costly check use. In this region the resource savings is larger, the smaller the average cost of stored value. Whether the resource savings from displacing checks outweighs the resource cost of displacing currency depends on the stored value cost function. It is straightforward to show that there is once again a cutoff value; if average stored value costs are below the cutoff, stored value is welfare-enhancing, while if average cost is greater than the cutoff, stored value reduces welfare.

### A Model with Costs of Handling Currency

This section describes an extension of the model in which handling currency is costly to merchants and shows that Proposition 1 also holds in this extended economy. (The same is true of an extended model in which handling currency is costly to shoppers, but that model is omitted here.) Suppose then that accepting currency as payment requires time-consuming effort on the part of merchants. For convenience, I will assume that the time requirement is proportional to the real value of currency handled and is equal to

$$\frac{\alpha p_t y_t^m(h)}{p_t},$$

where the parameter  $\alpha > 0$ . The feasibility condition (4) now becomes

$$y_t^m(h) + \alpha y_t^m(h) + y_t^s(h) + y_t^i(h) \leq n_t(h). \quad (4')$$

One unit of labor devoted to goods sold for currency now yields  $p_t/(1+\alpha)$  units of currency at the beginning of period  $t+1$ . One unit of labor devoted to goods sold to an issuer must provide the same yield, so in (5)  $y_t^i(h)$  is replaced with  $y_t^i(h)/(1+\alpha)$ . In the issuer's budget constraint,  $p_t c_t^i(z)$  is replaced by  $p_t c_t^i(z)/(1+\alpha)$ . The first-order condition from the issuer's problem becomes

$$r(z)(1+i) = \frac{\gamma(z)p_t}{(1+\alpha)p_t^s(z)}. \quad (11')$$

One unit of labor devoted to cash sales yields  $p_t/(1+\alpha)$  units of currency at  $t+1$ . Therefore (12) becomes

$$p_t = (1+\alpha)[1-r(z)](1+i)p_t^s(z). \quad (12')$$

The last two terms in the household's budget constraint (5) now simplify to  $p_t n_t(h)/(1+\alpha)$ .

Shoppers now use stored value if and only if  $(1+\alpha)(1+i) > [1+\gamma(z)]$ . Define  $\zeta(i)$  by  $(1+\alpha)(1+i) = (1+\gamma(\zeta(i)))$ . At this point I make a minor modification to the model. The preferences of issuers are altered so that goods from different locations are perfect substitutes:

$$c_t^i = \int_0^1 c_t^j(z) dz.$$

With this modification, the feasibility condition for this model simplifies to

$$n = c\{(1+\alpha)[1-\zeta(i)] + \Gamma(i)\}, \quad (16')$$

reflecting the fact that both currency handling costs and stored value costs affect the aggregate resource constraint. The first-order condition for this model is

$$\frac{u_1(c, 1-n)}{u_2(c, 1-n)} = (1+\alpha)\{1 + [1-\zeta(i)]i + \Gamma(i)\} \equiv q. \quad (15')$$

Once again the optimal monetary policy is the Friedman rule, but now stored value circulates even when the nominal interest rate is zero, since in some applications stored value is less costly than currency [ $\gamma(z) < \alpha$ ]. It is easy to demonstrate that Proposition 1 holds in this economy as well: for any fixed positive nominal interest rate there exists a cutoff  $\Gamma^*$  such that the stored value economy Pareto-dominates the economy without stored value if and only if  $\Gamma(i) < \Gamma^*$ .

---



---

## REFERENCES

Aiyagari, S. Rao, Toni Braun, and Zvi Eckstein. "Transactions Services, Inflation, and Welfare." Manuscript. 1995.

- Blinder, Alan. Statement before the Subcommittee on Domestic and International Monetary Policy of the Committee on Banking and Financial Services, U.S. House of Representatives, October 11, 1995.
- Board of Governors of the Federal Reserve System. *80th Annual Report*, 1993. Washington: Board of Governors, 1994a.
- \_\_\_\_\_. *PACS Expense Report*, 1993. Washington: Board of Governors, 1994b.
- Bryant, John, and Neil Wallace. "A Price Discrimination Analysis of Monetary Policy," *Review of Economic Studies*, vol. 51 (April 1984), pp. 279–88.
- Dotsey, Michael, and Peter Ireland. "The Welfare Cost of Inflation in General Equilibrium." Manuscript. 1995.
- English, William B. "Inflation and Financial Sector Size," Federal Reserve Board Finance and Economics Discussion Series No. 96–16 (April 1996).
- Friedman, Milton. "The Optimum Quantity of Money," in *The Optimum Quantity of Money and Other Essays*. Chicago: Aldine, 1969.
- \_\_\_\_\_. *A Program for Monetary Stability*. New York: Fordham Univ. Press, 1960.
- Ireland, Peter N. "Money and Growth: An Alternative Approach," *American Economic Review*, vol. 84 (March 1994), pp. 47–65.
- Kiyotaki, Nobuhiro, and Randall Wright. "On Money as a Medium of Exchange," *Journal of Political Economy*, vol. 97 (August 1989), pp. 927–54.
- Lacker, Jeffrey M. "Should We Subsidize the Use of Currency?" Federal Reserve Bank of Richmond *Economic Quarterly*, vol. 79 (Winter 1993), pp. 47–73.
- \_\_\_\_\_, and Stacey L. Schreft. "Money and Credit as Means of Payment," *Journal of Monetary Economics*, vol. 38 (August 1996), pp. 3–23.
- Laster, David, and John Wenninger. "Policy Issues Raised by Electronic Money." Paper presented to the Conference on Digital Cash and Electronic Money, held at the Columbia Business School, April 21, 1995.
- Lucas, Robert E., Jr., and Nancy L. Stokey. "Optimal Fiscal and Monetary Policy in an Economy without Capital," *Journal of Monetary Economics*, vol. 12 (July 1983), pp. 55–93.
- Marquis, Milton H., and Kevin L. Reffett. "New Technology Spillovers into the Payment System," *Economic Journal*, vol. 104 (September 1994), pp. 1123–38.
- \_\_\_\_\_. "Capital in the Payments System," *Economica*, vol. 59 (August 1992), pp. 351–64.



- Prescott, Edward C. "A Multiple Means of Payment Model," in William A. Barnett and Kenneth J. Singleton, eds., *New Approaches to Monetary Economics*. Cambridge: Cambridge Univ. Press, 1987.
- Rolnick, Arthur J., and Warren E. Weber. "The Causes of Free Bank Failures: A Detailed Examination," *Journal of Monetary Economics*, vol. 14 (November 1984), pp. 267–92.
- \_\_\_\_\_. "New Evidence on the Free Banking Era," *American Economic Review*, vol. 73 (December 1983), pp. 1080–91.
- Schreft, Stacey L. "Welfare-Improving Credit Controls," *Journal of Monetary Economics*, vol. 30 (October 1992a), pp. 57–72.
- \_\_\_\_\_. "Transaction Costs and the Use of Cash and Credit," *Economic Theory*, vol. 2 (1992b), pp. 283–96.
- Townsend, Robert M. "Currency and Credit in a Private Information Economy," *Journal of Political Economy*, vol. 97 (December 1989), pp. 1323–44.
- \_\_\_\_\_. "Economic Organization with Limited Communication," *American Economic Review*, vol. 77 (December 1987), pp. 954–71.
- \_\_\_\_\_. "Financial Structures as Communications Systems," in Colin Lawrence and Robert P. Shay, eds., *Technological Innovation, Regulation, and the Monetary Economy*. Cambridge, Mass.: Ballinger, 1986.
- Wallace, Neil. "The Impact of New Payment Technologies: A Macro View," in Colin Lawrence and Robert P. Shay, eds., *Technological Innovation, Regulation, and the Monetary Economy*. Cambridge, Mass.: Ballinger, 1986.
- \_\_\_\_\_. "A Legal Restrictions Theory of the Demand for 'Money' and the Role of Monetary Policy," *Federal Reserve Bank of Minneapolis Quarterly Review*, vol. 7 (Winter 1983), pp. 1–7.
- Whitesell, William C. "Deposit Banks and the Market for Payment Media," *Journal of Money, Credit, and Banking*, vol. 24 (November 1992), pp. 483–98.
- Williamson, Stephen D. "Laissez-Faire Banking and Circulating Media of Exchange," *Journal of Financial Intermediation*, vol. 2 (June 1992), pp. 134–67.