A WEEKLY RATIONAL EXPECTATIONS MODEL OF THE NONBORROWED RESERVE OPERATING PROCEDURE

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EDITOR'S NOTE : *This paper, although dealing with an operating procedure no longer employed by the Federal Reserve, is offered for its potential historical and analytical interest to monetary scholars. Note also that the paper's analysis may become relevant once more should the Fed again choose to target nonborrowed reserves in a future inflationary period.*

I.
INTRODUCTION

Of the many studies analyzing the Federal Reserve's post-October 6, 1979 nonborrowed reserve (NBR) operating procedure, none has focused on weekly money market dynamics under rational expectations.[1] This paper employs the rational expectations assumption in an explicit institutional model of the NBR procedure. The analysis is positive rather than normative, isolating the policy elements that comprise the procedure and investigating their dynamic interaction.

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[1] See Anderson and Rasche (1982), Avery and Kwast (1982), Axilrod (1981), Axilrod and Lindsey (1981), Bryant (1982), Farr and Porter (1982), Jones (1982), Levin and Meek (1981), Lindsey (1982), Lindsey and others (1981, 1984), Meek (1982), Poole (1982), Santomero (1983), Tinsley and others (1981, 1982A, 1982B), and Walsh (1982). The period under study extends from the fall of 1979 to the latter part of 1982 when the FOMC began to downplay the role of Ml in the targeting process citing uncertainties surrounding the behavior of this narrow aggregate.

The NBR operating procedure involved the interaction of three Federal Reserve policies: discount window administration, weekly nonborrowed reserve provision, and lagged reserve requirements. The model incorporates lagged reserve requirements in a straightforward way. It incorporates the characterization of Federal Reserve discount window administration and the associated optimization model of bank borrowing developed by Goodfriend (1983). A description given by Meek (1982) provides the basis for the model's characterization of nonborrowed reserve provision. Thus, the paper analyzes a stylized money market model whose main components capture the essential features of these three important aspects of monetary policy.

Modeling the NBR operating procedure requires using a model with both backward and forward looking dynamics. For instance, banks know that discount window administrative pressure increases the longer a given stay "in the window." Hence, current bank borrowing depends on both past borrowing and expected future borrowing since banks know that any borrowing today will, through informal Reserve Bank frequency guidelines, increase future borrowing costs. Concern for the future and the past also plays a role in the weekly provision of nonborrowed reserves. Within a given month, planned weekly nonborrowed reserve provision keeps current and projected weekly discount window borrowing roughly equal. For example, when new money stock numbers become available, the forecast of the following weeks' total reserve demand is updated. Simultaneously current week nonborrowed reserve provision and the planned future path for nonborrowed reserves are also adjusted so that planned borrowing over the remainder of the month is constant. Intertemporal considerations such as these complicate the analysis of the NBR operating procedure.

An adequate characterization of the nonborrowed reserve operating procedure requires a model with at least seven equations. Until recently, it was hard to find rational expectations solutions for systems of this size. We employ a new procedure developed by Anderson and Moore (1983, 1985) to efficiently solve the model. The primary goal of the paper is to use this solution technique to discover dynamic properties of the NBR monetary control procedure that have not been recognized before.

Section II presents the motivation and description of Federal Reserve discount window and nonborrowed reserve provision policy, as well as other standard money market model equations. Section III discusses the solution technique. We present and discuss plausible numerical values for the structural parameters in Section IV. Section V lays out the component policies and their dynamic properties. Each component of the NBR operating procedure responds in its own way to a disturbance. Section VI contains some discussion of the NBR operating procedure as a whole. A brief summary of results concludes the paper.

II.

A MODEL OF THE NONBORROWED RESERVE OPERATING PROCEDURE

In this section we develop two models. The first is a target generating model which determines the Federal Reserve's monthly average nonborrowed reserve target. The second is a weekly money market model. We link the two models together with a reduced-form equation from the targeting model which determines the provision of monthly average nonborrowed reserves as a function of the observable reduced-form variables in the targeting model. Week-by-week nonborrowed reserve provision is implemented in the money market model by the New York Reserve Bank Trading Desk under the assumption that it, along with other market participants, has rational expectations. We represent Federal Reserve policy in the model by (1) administration of the discount window, (2) the monthly average targeting procedure, (3) a gradual reentry path for the money stock in the targeting model, (4) assumptions in the targeting model concerning the demand for discount window borrowing, and (5) the Desk's imposition of a steady borrowing restriction in constructing the intramonth nonborrowed reserve path.

A. Discount Window Administration[2]

The System administers an effective form of nonprice rationing at the discount window. Regulation A states the condition under which a bank is entitled to "adjustment credit" at the discount window:

> Federal Reserve credit is available on a short-term basis to a depository institution under such rules as may be prescribed to assist the institution, to the extent appropriate, in meeting temporary requirements for funds, or to cushion more persistent outflows of funds pending an orderly adjustment of the institution's assets and liabilities.8

The regulation clearly indicates that bank borrowing should be of limited duration. The Report of the System Committee on the Discount and Discount Rate Mechanism (1954) also states that "the duration of borrowing [is] to be used to establish a rebuttable presumption that borrowing [is] for an inappropriate purpose."[4]

Reserve Banks have set up informal guidelines for administering their discount windows using duration as a measure of appropriateness. Although informal and not strictly followed, these guidelines are one means of triggering Reserve Bank contracts with senior officials of banks where discount window borrowing has been outstanding for sometime. In general, the guidelines imply progressively heavier pressure on banks the more lengthy a given stay in the window.

The existence of nonprice costs, frequency guidelines, and some degree of heterogeneity in discount window administration across Reserve Banks make it difficult to model discount window policy. However, rather than attempting to consider each of these complicating features here, we focus on the main aspect of the policy, progressive pressure.

The stylized model of discount window administrative pressure employed here has two components. First, the perceived marginal cost of borrowing rises with borrowing in the current week. Second, given the current week's volume of borrowing, the marginal

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[2] The analysis considered here is based on the model developed by Goodfriend (1983).

[3] Federal Reserve Board Rules and Regulations, Regulation A (as adopted effective September 1, 1980), sec. 201.3, par. a. Regulation A also entitles depository institutions to get seasonal and other so-called extended credit. Such borrowing is ignored throughout this paper. A good discussion of discount window administration is found in Board of Governors of the Federal Reserve System (September 1980).

[4] Board of Governors of the Federal Reserve System, Reappraisal of the Federal Reserve Discount Mechanism, vols. 1-3 (1971), p. 41.

cost of borrowing varies directly with the volume of borrowing in recent weeks. A simple linear-quadratic cost of borrowing function captures these two features of nonprice rationing:

$$(1) \quad C_t = \left[(\tfrac{1}{2})[B_{t-3}, B_{t-2}, B_{t-1}, B_t] \times \right.$$

$$\begin{bmatrix} 0 & 0 & 0 & c_4 \\ 0 & 0 & 0 & c_3 \\ 0 & 0 & 0 & c_2 \\ c_4 & c_3 & c_2 & c_1 \end{bmatrix} \begin{bmatrix} B_{t-3} \\ B_{t-2} \\ B_{t-1} \\ B_t \end{bmatrix} + dB_t$$

$$c_1, c_2, c_3, c_4 > 0,$$

where $B \equiv$ weekly discount window borrowing, $d \equiv$ the Federal Reserve discount rate.[6]

This functional form has several useful characteristics. First, the cost is zero when current borrowing, B_t, is zero. Second, the marginal cost of borrowing is positive and rises with B_t. Third, at any volume of current borrowing the marginal cost of borrowing varies directly with the volume of borrowing in the past three weeks. Finally, the marginal cost of current borrowing moves one-for-one with the current discount rate.

A bank will borrow in the current period (period t) until the marginal cost of an additional dollar of current borrowing just equals the marginal benefit. Differentiating the cost function with respect to B_t yields the first component of the current cost of an additional dollar of discount window borrowing

$$(2) \quad c_4B_{t-3} + c_3B_{t-2} + c_2B_{t-1} + c_1B_t + d.$$

Administrative pressure causes this component of the current marginal cost to be positive even if $B_{t-1} = B_{t-2} = B_{t-3} = 0$. This component rises with B_{t-1}, B_{t-2}, and B_{t-3} because the nonprice rationing mechanism makes the current marginal cost of borrowing depend positively on three lags of borrowing.

In rationally assessing the cost of additional current borrowing, a bank must also consider that current borrowing raises the marginal cost of borrowing in the future through the nonprice rationing mechanism. Specifically, in calculating its marginal cost of current borrowing the bank must include the present discounted value of the next three period's increased marginal cost of borrowing due to an extra

unit of current borrowing. Updating the cost function and differentiating with respect to B_t yields the second component of the current cost of an additional dollar of discount window borrowing,

$$(3) \quad bc_2B_{t+1} + b^2c_3B_{t+2} + b^3c_4B_{t+3}$$

where $b \equiv$ a constant discount factor. Note that this component of the current marginal cost is zero if the next three period's borrowing turns out to be zero. The inclusive marginal cost of borrowing is the sum of both components (2) and (3).

The current marginal benefit of an extra unit of discount window borrowing is the opportunity cost of obtaining the funds in the Federal funds market, i.e., the current Federal funds rate, f_t.

A bank maximizes the present discounted value of profits (the net benefit of borrowing at the discount window) by raising B_t to the point where the inclusive marginal cost of B_t borrowing just equals the marginal opportunity cost. Satisfying this condition, known as the Euler equation, is a necessary condition for B_t to be optimal. The Euler equation for the bank borrowing problem is

$$(4) \quad b^3c_4B_{t+3} + b^2c_3B_{t+2} + bc_2B_{t+1} + c_1B_t + c_2B_{t-1} + c_3B_{t-2} + c_4B_{t-3} = f_t - d$$

or more simply

$$(4a) \quad \sum_{i=-3}^{3} \phi_iB_{t+i} = f_t - d.[6]$$

Equation (4) is a necessary condition for optimal bank borrowing from the discount window when the cost of borrowing function (1) characterizes Federal Reserve discount window administration. Equation (4) is not an operational demand function since it does not express B_t as a function of the period t spread between the Federal funds rate and the discount rate and variables in the bank's information set at time t. Transforming (4) into a demand function would require replacing B_{t+1}, B_{t+2}, and B_{t+3} with rational forecasts based on period t information. But since rational forecasts depend on the entire structure of the model, they can only be acquired by solving the model as a whole.

Equation (4) is not a structural model equation either because its form depends on institutional rules

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[5] The use of a three-week lag was chosen arbitrarily to capture the effect of some lags without making the model unmanageable.

[6] We implicitly assume that $B \gtreqless 0$ all along the solution path.

established by the Federal Reserve: the equation's leads, lags, and coefficient values depend on the administration of the discount window. Nevertheless, (4) does contain all the restrictions on borrowing and the funds rate-discount rate spread implied by bank profit maximization in response to the Federal Reserve's discount window administration. As such (4) serves as a fundamental model equation.

More generally, the use of progressive pressure by the Federal Reserve to raise the perceived cost of discount window borrowing appears to be a reasonable policy for the Federal Reserve in its role as a lender of last resort. The policy provides relatively inexpensive reserve credit to cushion banks in periods of unanticipated funds rate increases, while providing an automatic inducement for banks to gradually wean themselves from the discount window. But because the policy necessarily makes past and expected future borrowing volume influence current borrowing demand, it introduces a dynamic element into the model solution.

B. A Model of the Monthly Average Targeting Procedure

Having concluded that the Federal funds rate was an unreliable instrument for controlling the money stock, the FOMC adopted a "reserve targeting" operating strategy in its anti-inflation program announced on October 6, 1979.[7] In principal, NBR targeting has three attractive features. First, it promises better monetary control, and with it better control of inflation. Second, it requires less detailed information about the relation between the level of short-term interest rates and money growth. That information requirement was seen as one of the major difficulties with using the funds rate as the instrument of monetary control. Third, by requiring the Desk to divorce itself from direct day-to-day control of the funds rate, the procedure is valuable in shifting the responsibility for interest rate movements from the Federal Reserve to the market. This separation in turn makes it easier for the Federal Reserve to concentrate on monetary control and long-run price stability.

The simplest reserve targeting strategy would have been to follow a strict weekly target path for total reserves consistent with the money stock moving along a desired noninflationary path. However, actual reserve targeting differed from this simple strategy for four reasons.

First, under the then existing system of lagged reserve requirements with limited carryover, total reserve demand in a given reserve statement week was essentially predetermined to support deposits held during a previous reserve computation period. Consequently, if required reserves fell below a total reserve target in a given week, reserve market equilibrium would have had to be achieved by a funds rate fall sufficiently large to induce the banking system to willingly absorb the extra reserves supplied as excess reserves. Conversely, if required reserves were above the total reserve target, the reserve market could only clear after the funds rate rose to the point where it either exceeded the cost of going deficient, or else drew sufficient currency out of circulation. Short-run monetary control would not only have been difficult using this type of procedure, but the Federal Reserve would have had to tolerate potentially large funds rate fluctuations to implement such a strategy. As a result, the FOMC chose to implement its reserve targeting strategy by targeting nonborrowed rather than total reserves.[8] With nonborrowed reserve targeting, reserve market clearing was to be achieved with less funds rate volatility because reserve demand would be partially accommodated by discount window borrowing.

Second, the Desk constructed the weekly nonborrowed reserve target path to be consistent with a projected monthly average of weekly money stock numbers.[9] The motivation for targeting monthly average rather than weekly money seemed reasonable given the apparent high degree of noise in the weekly money series.[10]

Third, monthly average money stock targeting also provided latitude for adjusting the intramonth nonborrowed reserve path to achieve a secondary objective that the Federal Reserve thought desirable. Following a "steady borrowing" objective, the Desk chose a weekly NBR path, consistent with a predetermined monthly average NBR, so that projected discount window borrowing would remain constant over the remainder of the month.[11] Given a stable demand for discount window borrowing, the Federal Reserve can approximately stabilize the funds rate-discount rate spread by stabilizing the level of bor-

[7] See Board of Governors (February 19, 1980).

[8] A case for targeting total reserves following the 1984 move to contemporaneous reserve requirements is contained in Goodfriend (1984).

[9] See Meek (1982, p. 102).

[10] See Pierce (1981).

[11] See Meek (1982, pp. 102-3).

rowing that it forces upon the banking system. As such, the preference for intramonth steady borrowing reflected a desire for funds rate smoothing.

The fourth and last reason that the reserve targeting strategy differed from strict weekly total reserve targeting is that the monthly average money stock target itself was not tied rigidly to a steady state path. When the money stock departed from the steady state target path, the Federal Reserve targeted it to return to the longer run path gradually over time.[12] The gradual "reentry path" for monthly average money was apparently motivated by a desire to accommodate the demand for money over periods of time longer than a few weeks to further smooth interest rates.

Table I presents our formalization of how the Federal Reserve determined the monthly average nonborrowed reserve target. Our model is recursive and begins with an equation describing the money stock target generating process. The equation is motivated by a simple characterization of the process that makes the monthly average target for a given month some fraction of the gap between the previous month's monthly average money stock realization and the steady state target.[13] In a weekly context this can be modeled as

$$(5) \quad M_k^* = \lambda M_{k-1}^*, \quad 0 \leq \lambda < 1$$

$$(6) \quad \text{and for } k < t \quad M_k^* = M_k.$$

The weekly money target for week k (M_k^*) set in week t decays toward the long-run target at a rate of $1 - \lambda$ per week. However, when speaking of the reentry rate throughout the paper we shall simply be referring to λ. The initial value for the target path is the realized week t-1 money stock. All variables in the paper are deviations from steady state values except the discount rate which is held constant by assumption.

The first step in constructing a monthly average nonborrowed reserve target for a month beginning with week t is to calculate the weekly money stock target path implied by (5) in terms of the realized money stock in week t-1. (For analytic simplicity we assume a month has three weeks.) Given a target path for the money stock, the Federal Reserve calculates the Federal funds rate path that is consistent with this money path. The Federal Reserve's calcu-

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Table I

EQUATIONS DESCRIBING THE FEDERAL RESERVE'S MONTHLY AVERAGE TARGET GENERATING PROCEDURE

$$(5) \quad M_k^* = \lambda M_{k-1}^*$$

$$(7) \quad M_k^* = \delta M_{k-1}^* + a_1 \sum_{i=0}^{3} b^i f_{t+1}^*$$

$$(8) \quad \left(\sum_{i=-3}^{3} \phi_i \right) B_k^* = f_k^* - d$$

$$(11) \quad ER_k^* = b_1 f_k^*$$

$$(10) \quad RR_k^* = \rho M_{k-2}^*$$

$$(9) \quad RR_k^* + ER_k^* = NBR_k^* + B_k^*$$

$$(13) \quad \overline{NBR}_k = (\tfrac{1}{3}) \sum_{i=0}^{2} NBR_{k+1}^*$$

Initial conditions:

$$\text{for } k < t, \quad M_k^* = M_k$$

lation is necessarily based on its best estimate of the public's weekly demand for money, which we write as

$$(7) \quad M_k^* = \delta M_{k-1}^* + a_1 \sum_{i=0}^{3} b^i f_{k+i}^*$$

$$0 < \delta < 1, \quad a_1 < 0$$

where f* is the Federal funds rate that is consistent with the target path for money.

Equation (7) states that weekly money demand is positively related to last week's demand and negatively related to a discounted sum of current and future funds rates. This equation embodies the notion that weekly money demand depends on a longer term rate than a weekly rate. Instead of specifying this longer term rate separately in a term structure equation, it is embedded in (7) directly. Again note that equation (7) like equation (4) is not an operational demand function since it does not express M_k^* solely as a function of variables in the public's or the Federal Reserve's information set in week k.

By substituting the weekly money stock target path from (5) into (7) one derives a weekly funds rate path required to induce the public to hold the targeted quantities of money. Interestingly, if the Federal Reserve had a preference for a smooth funds

rate path, it could chose A equal to its best guess of δ and thereby generate a flat projected funds rate path.

With the required funds rate path in hand, the Federal Reserve can then set out to construct reserve paths to achieve the funds rate target. To do this the Federal Reserve needs a view of the relationship between the volume of discount window borrowing and the current spread between the funds rate and the discount rate. For now, suppose that the Fed views this relationship as purely contemporaneous,

$$(8) \quad B_k^* = h(f_k^* - d)$$

$$\text{where } h \equiv 1/(\sum_{i=-3}^{3} \phi_i) > 0.$$

We consider the intertemporal version of this equation later.

By (8) there is a particular borrowing path associated with the required funds rate path. In order to "force" this required path for borrowing, the Federal Reserve first projects total reserve demand. To do this it uses the reserve accounting identity

$$(9) \quad RR_k^* + ER_k^* \equiv NBR_k^* + B_k^*$$

where $RR^* \equiv$ required reserves, $ER^* \equiv$ excess reserves, $NBR^* \equiv$ nonborrowed reserves, and $B^* \equiv$ borrowed reserves. In calculating total reserve demand it uses the reserve requirement rule

$$(10) \quad RR_k^* = \rho \, M_{k-2}^*$$

where $\rho \equiv$ the reserve requirement ratio, together with its best estimate of the weekly demand for excess reserves, which is assumed to be

$$(11) \quad ER_k^* = b_1 f_k^* \qquad b_1 < 0.$$

Then using equations (8) through (11), the Federal Reserve derives a path for nonborrowed reserves that generates the borrowing path and thereby the funds rate path required to hit the money stock targets implied by (5). Letting NBR* represent these weekly targets, the NBR*'s constructed in week t for the month beginning with week t are

$$(12) \quad NBR_t^* = \rho \, M_{t-2} + (b_1 - h)f_t^*$$
$$NBR_{t+1}^* = \rho \, M_{t-1} + (b_1 - h)f_{t+1}^*$$
$$NBR_{t+2}^* = \rho \, M_t^* + (b_1 - h)f_{t+2}^*$$

where the discount rate has been assumed to remain at its steady state value, so d = 0. The monthly average nonborrowed reserve target for the month beginning with week t then becomes

$$(13) \quad \overline{NBR}_t = \tfrac{1}{3} \sum_{i=0}^{2} NBR_{t+i}^* .$$

As a consequence of lagged reserve requirements, the Federal Reserve can use observations on money realized in weeks t-2 and t-1 to calculate the required reserve component of the nonborrowed reserve target for the first two weeks of the month. But the Fed must base its projection of required reserves for the last week of the month on the week t money stock target, M_t^*. This explains the use of M_t^* in place of M_t in (12). Hence, in our model the Desk's nonborrowed reserve provision for a given week is determined by relevant information available to the Federal Reserve, which in this case is the observed money stock in the two previous weeks.

C. Weekly Nonborrowed Reserve Provision by the Trading Desk

Levin and Meek state (1981, pp. 7-8) :

The Desk begins each intermeeting period with a path for nonborrowed reserves (the total reserve path estimated by the Board staff less the Committee's initial assumption for borrowing at the discount window). Each week, as new information becomes available, senior Board staff and the Account Management review, and revise, if appropriate, the reserve paths to maintain their consistency with the Committee's aggregate objectives. Then the Desk must translate the reserve paths into weekly operating objectives for nonborrowed reserves. This is done in the following way: First, the staff projects the demand for total reserves-that is, required reserves based on actual or estimated deposits plus excess reserves. Second, the average projected demand for total reserves over the period is compared to the average nonborrowed reserve path over the period. This, given actual levels of borrowing in earlier weeks, provides an estimate of average borrowing over the remaining weeks if the average nonborrowed reserve path is to be achieved. Finally, this steady level of borrowing is subtracted from the projected demand for total reserves in each of the remaining weeks to give a series of weekly nonborrowed reserve objectives. [Emphasis added.]

Formally, we model implementation of the Desk's steady borrowing restriction for a week t in the middle of a reserve targeting month beginning with week t-l by choosing $\widetilde{B}_t$ to satisfy

$$(14) \quad \tfrac{1}{3} \sum_{i=-1}^{1} (RR_{t+i} + ER_{t+i}) -$$
$$\tfrac{1}{3} (B_{t-1} + 2\widetilde{B}_t) = \overline{NBR}_{t-1} .^{[14]}$$

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[14] For a detailed example of the steady borrowing restriction, see Meek (1982, pp. 102-3).

Using equation (14), with projections of RR and ER for periods t and t+1 conditioned on information in period t, the Desk calculates the equal levels of borrowing for weeks t and t+1 that satisfy the monthly average nonborrowed reserve target inherited from week t-l. That is, the Desk constructs the nonborrowed reserve path for weeks t and t+1 using the "steady borrowing" restriction $\tilde{B}_t = \tilde{B}_{t+1}$. Thus, week t nonborrowed reserve provision is

$$(15) \quad NBR_t = RR_t + ER_t - \tilde{B}_t$$

where RR_t and ER_t are values expected at the beginning of period t.

We solve the model as if each week t were the middle week in a three-week targeting month. Consequently, in each week t the Desk employs a monthly average nonborrowed reserve target constructed in week t-l, together with equations (14) and (15), and forecasts of relevant reserve demands for the remainder of the targeting month, to determine nonborrowed reserve provision for that week. This solution procedure, in effect, operates as if the Federal Reserve never reaches the last week of a monthly average targeting month. On the face of it, such an abstraction seems to miss an important constraint embodied in monthly average targeting: that in the last week of a targeting month, nonborrowed reserves must be set to hit the monthly average target regardless of whether the associated weekly borrowing and funds rate are expected to be higher or lower than in subsequent weeks. However, in practice, when the Federal Reserve reached the last week of a monthly reserve targeting period, it often abandoned its monthly average nonborrowed reserve target to make borrowing in the current week equal to the expected initial borrowing objective for the following monthly targeting period.[15] In other words, maintaining continuity of borrowing and a flat funds rate forecast profile seemed to override hitting the monthly aver-

age nonborrowed reserve target at the end of a targeting month. That is why we always model the Federal Reserve in the middle of a targeting month.[16]

D. The Basic Money Market Model Equations

The preceding discussion of nonborrowed reserve provision involved a specification of the money market as the Federal Reserve believes it to be. For example, we postulated that the Federal Reserve believed the relation between the volume of discount window borrowing and the spread between the funds rate and the discount rate to be purely contemporaneous. In this subsection we present the equations describing actual money market model behavior. The basic set of money market model equations with variables, except for the Federal Reserve discount rate, written in deviation from steady state values are the money demand equation

$$(16) \quad M_t = \delta M_{t-1} + a_1 \sum_{i=0}^{3} b^i f_{t+i} \ ;$$

the discount window borrowing equation (4a)

$$(17) \quad \sum_{i=-3}^{3} \phi_i B_{t+i} = f_t - d \ ;$$

the demand for excess reserves

$$(18) \quad ER_t = b_1 f_t \ ;$$

the demand for required reserves

$$(19) \quad RR_t = \rho M_{t-2} \ ;$$

and the reserve accounting identity

$$(20) \quad ER_t + RR_t \equiv NBR_t + B_t \ .$$

As explained in Section II. B above, for each week t our representation of the Federal Reserve's targeting procedure yields a monthly average NBR target, NBR_t, based on the previous two week's realizations of money:

$$(21) \quad \overline{NBR}_t = g(M_{t-1}, M_{t-2}) \ .$$

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[15] Levin and Meek (1981, p. 20) state:

> In accounting for deviations between actual and path values for nonborrowed reserves, it is useful to distinguish between accepted or "intentional" misses and unintentional misses. Accepted or intentional misses, which accounted for over two-thirds of the deviations, represented decisions to tolerate or even aim for reserve supplies either above or below average path values. They arose from a variety of considerations, but mainly reflected deviations from expectations for borrowing in the final week of a reserve period and a desire to maintain continuity in the degree of adjustment pressure on the banks in the transition from one control period to the next around the time of FOMC meetings. [Emphasis added.]

[16] Putting the Desk in the middle of the targeting month is also convenient for computational reasons. If each week of the targeting month had to be modeled individually, the dimensionality of the model would increase threefold.

In any given week t the Desk operates with the monthly average NBR target determined in the previous week. So week t nonborrowed reserve supply is determined, from equations (14) and (15), to satisfy inherited $\overline{NBR}_{t-1}$ and the Desk's steady borrowing restriction

$$(22) \quad NBR_t = RR_t + ER_t -$$

$$\frac{1}{2} [\sum_{i=-1}^{1} (RR_{t+i} + ER_{t+i}) -$$

$$\overline{NBR}_{t-1} - B_{t-1}] .$$

The set of money market model equations (16) through (22) can be solved for the money, funds rate, and reserves generating processes. The major difficulty in obtaining a solution is that forecasts of variables not yet known in week t play an important role in the money demand, borrowing, and reserve provision equations. In solving the model, week t forecasts of the public and the Desk are the same as the model's forecasts conditioned on information available in week t. In other words, this solution procedure assumes that both the public and the Desk have rational expectations.

III.
SOLUTION TECHNIQUE

Anderson and Moore (1983, 1985) specify in detail the procedure used to solve the model. They consider a general linear model whose solution for period t depends on the solution for periods both prior and subsequent to t :

$$(23) \quad \sum_{i=-\tau}^{\theta} H_i X_{t+i} = 0, \quad t \geqq 0 .$$

The lenght of the maximum lag and lead in the model, τ and θ, are both positive, and X is an L dimensional vector. The initial conditions

$$(24) \quad X_i = \overline{X}_i, \quad i < 0,$$

are given by history. They assume that the coefficient matrices H_i have the saddlepoint property assumptions a and b:

(a) The origin is the unique steady state of equation (23). That is, if $(\sum_{i=-\tau}^{\theta} H_i) X = 0,$ then X = 0.

(b) For any set of initial conditions $\overline{X}_i$, i < 0, equation (23) has a unique solution sequence X_t, $t \geqq 0$, converging to the steady state ; $\lim_{t \to \infty} X_t = 0.$

Anderson and Moore prove that any such model has a reduced form relating the unique stable solution sequence entirely to its history: there is a set of reduced-form coefficient matrices such that the unique stable solution to equation (23) can be written as

$$(25) \quad X_t = \sum_{i=-\tau}^{-1} B_i X_{t+i}, \quad t \geqq 0 .$$

The proof is constructive, displaying an efficient procedure for computing the reduced form of any model that has the saddlepoint property. Given numerical values for a model's parameters, the procedure either produces a reduced form of the model or indicates why a reduced form does not exist. In particular, a model can violate assumption b because it has (1) multiple stable solutions for any initial conditions or (2) a stable solution only for a restricted subset of feasible initial conditions.

Finally, as a formal matter, when assumption b is violated it is possible under some conditions to derive a reduced form that yields the fastest converging of the stable multiple solutions or the slowest diverging of the unstable solutions.

Our model of the nonborrowed reserve operating procedure requires two applications of the solution routine. The solution routine is applied first to the equations, listed in Table I, in our model of the Federal Reserve's monthly average targeting procedure. While in practice we solve them simultaneously, in princple they can be solved recursively: Solve the first equation for the targets, M_k^* ; solve the second equation for the funds rate path, f_k^*, consistent with the money path; solve the third for the borrowings path, B_k^*, consistent with the funds rate path; and so forth. On completion, the reduced form of the monthly average nonborrowed reserve target NBR_k is a function of the lagged variables in the targeting model, M_{k-1}^* and M_{k-2}^*,

$$(26) \quad \overline{NBR}_k = g(M_{k-1}^*, M_{k-2}^*) .$$

At this point we note that lagged values of M* are equal to deviations between lagged realized money and the Federal Reserve's steady state money stock target which is assumed to be constant ; so we can use

$(27)\ M_k^* = M_k \quad$ for $k < t$

to write

$(28)\ \overline{NBR}_t = g(M_{t-1}, M_{t-2})$.

This reduced-form NBR target generating equation then appears among the equations of the weekly money market model, listed in Table II. The money market model equations are fully simultaneous, and we Apply the solution procedure again to compute the reduced form of the model as a whole.

It is worth noting that the Federal Reserve's money stock target M* will generally differ from the model solution M for the following reasons. First, the Federal Reserve operates with information lags, so that it must set its instrument, in this case weekly nonborrowed reserves, prior to observing the weekly money stock realization. Second, because of the imposition of interest rate smoothing restrictions, the Federal Reserve's targeting procedure does not generally put the expected money stock generated by the money market model solution on the targeted money stock path.

Having computed the model's reduced form, we informally analyze its dynamic behavior by computing the response to a single disturbance. In these experiments we assume that agent's expectations formed in week t are based on information through week t-l. Furthermore, we assume that model variables in the

$(16)\ M_t = \delta M_{t-1} + a_1 \sum_{i=0}^{3} b^i f_{t+i}$

$(17)\ \sum_{i=-3}^{3} \phi_i B_{t+i} = f_t - d$

$(18)\ ER_t = b_1 f_t$

$(19)\ RR_t = \rho M_{t-2}$

$(20)\ RR_t + ER_t = NBR_t + B_t$

$(22)\ \overline{NBR}_{t-1} = (\tfrac{1}{3})[\ \sum_{i=-1}^{1} (RR_{t+i} + ER_{t+i}) -$

$\qquad\qquad B_{t-1} - 2B_t]$

$(21)\ \overline{NBR}_t = g(M_{t-1}, M_{t-2})$,

a reduced-form equation from the monthly average targeting submodel.

week of the disturbance have been at the steady state for as long as the longest lag in the model. Thus in the notation of equation (24),

$(29)\ X_i = 0, \quad i = -\tau, \dots, -1$.

Based on this information, agents forecast that the solution will remain at the steady state:

$(30)\ \widetilde{X}_t = 0, \quad t \geqq 0$.

We then subject the model equations to a single shock of ϵ_0 in week zero. Under our information assumptions the model variables solve

$(31)\ \quad H_0 X_0 = \epsilon_0$.

The lagged variables are actually zero, but the future variables are incorrectly expected to be zero because of the information lag. We subject the model to no further shocks so that agents' expectations are realized after week zero, and the impulse response is given by

$(32)\ X_t = \sum_{i=-\tau}^{-1} B_i X_{t+i}, \quad t > 0$

with initial conditions

$$X_0 = H_0^{-1} \epsilon_0 ,$$
$$(33)$$
$$X_i = 0 \quad i < 0 ,$$

where the B_i's are the reduced-form coefficients of the money market model.

Table III presents the parameter values that are chosen to represent the model. Our objective in assembling these numbers is to obtain ballpark figures in line with other work on the subject. Except for borrowing and the size of the lagged dependent variable in the money demand function, the values are quite typical and there is not much to discuss.

We assume that the shape of the borrowing cost function (1) is such that

$(34)\quad c_i = K c_{i+1}, \quad$ where $0 < K < 1$;

or in the notation of (4a)

$$\phi_i = \begin{cases} K^{|i|} c_4 & i \leqq 0 \\ (Kb)^i c_4 & i > 0 . \end{cases}$$

Table III

BASELINE MODEL PARAMETERS

Parameter	Symbol	Value	Appears in Equation	Source
long-run slope coefficient of borrowings	$1/(\sum_{i=-3}^{3} \phi_i)$	= .240 billion per percent per year	Fundamental borrowing (4a)	See text.
	$\phi_i = \begin{cases} \phi_0 K^i & i \leq 0 \\ \phi_0 (bK)^i & i > 0 \end{cases}$	K = .62		K chosen so that borrowing equation by itself is stable but near the region of instability, see text.
	ϕ_0	= 1.197		Value of ϕ_0 derived from previous two assumptions.
Reentry parameter λ		$= (.7^3)^{1/13} = .921$	Money targeting (5)	Translation of Tinsley, et al. (1981) reentry estimate of .7 on monthly data to weekly data.
lagged dependent variable δ		= exp[1n(.5)/13] = .948	Money demand (7)	Assumed lagged dependent variable of .5 in quarterly money demand equation. This estimate corresponds roughly to the lower limit in the 95 percent confidence interval estimate for demand deposits presented by Goldfeld (1973, p. 596).
Impact slope coefficient in money demand function	$a_1 = \dfrac{(1-\delta)}{(\Sigma b^i)} \dfrac{(M^*)}{f^*} (\eta_{m,f}) = \dfrac{(1-.948)(250)}{3.990 \times 8}(-.10) = -.04$			Assumed long-run money demand elasticity $(\eta_{m,f})$ equals - .10. This estimate is slightly below that obtained by Goldfeld for MI, Goldfeld (1973, Table -16, average of open market rate estimates).
Required reserve ratio ρ		= 39.6/250 = .1584	Required reserves (10)	Ratio of ballpark figures for required reserves and demand deposits.
Slope coefficient in excess reserves b_1		= - .00484 billion per percent per year	Excess reserves (11)	Tinsley (198 1).
Constant discount rate b		$= 1 - \dfrac{f^*}{5000} = 1 - \dfrac{8}{5000}$ = .9984 per week	Fundamental borrowing (4a) and money demand (7)	Derived from assumption that steady state interest rate = 8.0 percent per year and 50 weeks in a year.

That is, costs decline linearly from a peak at time t. From an analysis of the homogeneous difference equation associated with (4a) it can be shown that for values of K slightly above .62, the difference equation is unstable. That is, unless a fairly high percentage of the costs are concentrated contemporaneously at time t, the equation is unstable. For example, if K = 1, so that the weights have a uniform distribution, the equation is unstable. Setting K = .62 is thus useful for a study of the dynamic properties of the system.

We assumed that the reciprocal of the sum of the coefficients on borrowing in equation (4a) equals .240 billion dollars per percentage point spread between the discount rate and the Federal funds rate. Such a value is associated with a weekly impact coefficient on borrowing of $1/\phi_0 = .8354$ billion dollars per percentage point in the spread. This borrowing impact lies between the estimates that Levin and Meek (1981) report for 1972-74 and for the period from October 1979 to November 1980, respectively.

Finally, we have assumed that the lagged dependent variable in the money demand function is .5 for quarterly data. This number is somewhat less than most quarterly or monthly model estimates, but in line with much judgmental analysis of the relationship between interest rates and the demand for money. Since, as Goodfriend (198.5) has argued, there are no compelling theoretical reasons to justify any lag in the relationship, we chose an estimate that was deliberately on the low side of most econometric estimates.

V.
SOME PARTIAL POLICY EFFECTS

In this section, we examine model solutions to isolate the effect of individual components of the policies that make up the nonborrowed reserve operating procedure. We illustrate the effects by describing the response of the model to a one billion dollar (positive) shock to money demand. The disturbance becomes known to the Federal Reserve and market participants with a one-week data collection lag. Throughout the section, reserve requirements are taken to be lagged. The goal is to isolate successively the effects of progressive pressure discount window administration, monthly average NBR targeting, the steady borrowing restriction within a reserve targeting month, and pure weekly money targeting with a gradual reentry rate $\lambda > 0$. The model responses discussed in this section are illustrated in Figures 1 through 4.

A. Progressive Pressure Discount Window Administration

To focus on the effect of penalizing duration of borrowing at the discount window, we investigate the implications of a money demand shock in a model where nonborrowed reserves are fixed at their steady state value on a week-by-week basis. Since reserve requirements are lagged and aggregate money is not contemporaneously observable, on impact (in week zero) a money demand shock affects only the money stock. The funds rate, excess reserves, required reserves, and borrowings all remain at their steady state values in week zero. This delay illustrates the highly accommodative aspect of lagged reserve requirements. Lagged reserve accounting implies that even in week one when agents in the model observe the aggregate money stock increase, required reserve demand for that week will not change. Moreover, given the dominance of the lagged dependent variable in the money demand equation relative to the interest rate effect, money will remain high from week one onward. Thus banks see that their reserve demand will be above the steady state from week two onward. As a consequence of progressive pressure discount window administration, banks forecasting an increase in borrowing requirements become a little less willing to borrow in week one. Therefore, the reserve market clearing funds rate rises in week one. Using the calibration in Table III, the one-billion-dollar money demand shock causes the week one funds rate to rise by 21 basis points.

The money demand shock actually begins affecting reserve demand in week two. By assumption banks must meet these reserve requirements by borrowing the entire volume, .16 billion dollars, from the discount window. The forced increase in borrowing drives the funds rate up another 16 basis points. Following week two, bank borrowings continue to be above normal so that the progressive administrative pressure at the discount window continues to rise. This effect raises the funds rate that clears the reserve market in weeks three, four, and five. As a result of the three-week lag in the borrowing cost function, the maximum progressive pressure occurs after banks have borrowed heavily for three consecutive weeks, i.e., in week five. For our model calibration, the funds rate peaks in week five having risen 45 basis points above its steady state value. Following that, the funds rate gradually falls. By week ten it is back to 20 basis points above its steady state value. By this time the money stock is .21 billion dollars above its long-run target. This policy

produces an actual reentry rate for money of .86.[17] Money returns to steady state faster than our calibrated rate of .92 primarily because weekly nonborrowed reserves are fixed at their steady state value. A policy with more accommodative nonborrowed reserve provision would require less borrowing and yield lower interest rates. Lower interest rates, in turn, would produce slower convergence of money to its steady state.

In short, progressive pressure at the discount window may be said to delay the funds rate response relative to what would be produced if discount window administration were based exclusively on contemporaneous borrowing. In this model with three weeks of lags in the borrowing function, this policy delays the funds rate peak three weeks. If the Federal Reserve wishes to postpone the funds rate peak in response to a money demand shock, then progressive pressure at the discount window has value in doing so. However, progressive pressure also introduces oscillatory behavior into borrowing demand. Technically, progressive pressure induces either complex or negative roots into the model solution. The oscillation is simply a result of the fact that for a given funds rate-discount rate spread, progressive pressure makes abnormally high borrowing in one week cause borrowing demand in the following week to move below normal. The effect is present regardless of the number of lags in the borrowing cost function.

B. Monthly Average Nonborrowed Reserve Targeting

To isolate the effect of monthly average nonborrowed reserve targeting on the model solution, we assume that (1) nonborrowed reserves are supplied to hit their monthly average steady state value, so $NBR_{t-1} + NBR_t + NBR_{t+1} = 0$, and (2) progressive pressure is not a feature of discount window administration so that the structure of lags and leads does not matter, i.e., $(\Sigma \phi_i) B_t = f_t - d$. Since we model monthly average nonborrowed reserve targeting as if the Federal Reserve were always in the middle of a three-week month, in any given week t the Desk must target $NBR_t + NBR_{t+1}$ equal to the predetermined NBR_{t-1}. This constraint implies, in turn, that $NBR_{t+s} = NBR_t 1$, so that adhering to the monthly average target requires weekly nonbor-

rowed reserve provision to cycle. In other words, the model propagates the initial condition on NBR_{t-1} at period three forever.

Admittedly, the cycling feature of monthly average targeting is a consequence of the "rolling month" targeting assumption. Under "calendar month" targeting, the past becomes irrelevant at the beginning of each new calendar targeting month; so the propagation of the initial condition is truncated.

Unfortunately, the calendar targeting month, by having an actual last week of the month that the Federal Reserve must face up to, introduces another difficulty. As the month unfolds and nonborrowed reserve realizations accumulate, hitting monthly average nonborrowed reserves can require large temporary weekly movements in nonborrowed reserve supply which would produce large, temporary borrowed reserve and funds rate effects.

As an alternative to these two monthly averaging procedures, targeting could be done on a rolling month basis as above except that the Desk could always view itself as being at the beginning of a new monthly average targeting period. This procedure would have neither of the problems of the two previously discussed types of monthly average targeting. However, a little thought shows that this procedure isn't monthly average targeting at all. It would never make last week's nonborrowed reserve provision relevant to the choice of the current week's provision, nor would it ever make the Federal Reserve face up to the last week of a reserve targeting month. These are the two essential constraints implied by monthly average targeting.

C. Steady Borrowing with Monthly Average Reserve Targeting

In this subsection, we assume that the Desk still targets monthly average nonborrowed reserves at its steady state value, so NBR = 0, and that progressive pressure is not a feature of discount window administration, so $(\Sigma \phi_i) B_t = f_t - d$. However, in contrast to Section V. B, we suppose that the Desk imposes a steady borrowing restriction in constructing its weekly NBR reserve path within a given reserve targeting month. Mathematically, this means that nonborrowed reserves are determined by equation (22) with $\overline{NBR}_{t-1} = 0$.

Since the Desk cannot observe money contemporaneously and reserve requirements are lagged, the money demand shock in week zero affects only the money stock. When the week zero aggregate money

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[17]This result hinges critically on the arbitrary choice of a three-week lag in borrowing cost function (1). Ten weeks after the shock the simulated money stock is $(.86)^{10} M_0$.

stock increase becomes known one week later, the Desk can forecast a .16 billion dollar required reserve demand increase in week two because of lagged reserve requirements. If the intramonth nonborrowed reserve path were not adjusted, the Desk could then expect a large jump in week two discount window borrowing. But in order to keep planned borrowing flat over the remainder of the month, it pulls NBR_1 down and raises planned NBR_2 by half the projected week two increase in required reserve demand, .08 billion dollars. Consequently, week one borrowing rises by .08 billion dollars. The associated week one funds rate rise is 32 basis points.

Equation (14), with t=2 and NBR = 0, determines the volume of borrowing that policy induces in week two. Lagged borrowing, B_1, has risen by .08 billion dollars, but now the Desk expects total reserve demand to be about .16 billion dollars higher in both the current and in the last week of the current reserve targeting month. On net, this means that the Desk must raise B_2 by an additional .04 billion dollars to maintain steady borrowing. The result is that the funds rate rises by another 12 basis points in week two.

Reasoning by analogy, in week three the volume of borrowing induced by this policy rises again and consequently the funds rate rises again, this time by about 19 basis points to its peak about 63 basis points above its steady state value. In week four, the funds rate takes a relatively large drop of about 20 basis points. This decline occurs because in the fourth week after the shock, monthly total reserve demand remains approximately unchanged, but lagged borrowing climbs to its maximum in this week. As can be seen in equation (14) for t=4, this last jump causes the Desk to reduce B_4 and planned B_5 to achieve the monthly average reserve target centered on week four.

The remaining adjustment of the funds rate, borrowing, and money back to the steady state occurs gradually and smoothly as a result of the operation of the lagged dependent variable in the money demand equation. By week ten the funds rate is back at about 18 basis points above its steady state value, and money is about .21 billion dollars above its steady state. In fact, as in all these examples, given the relatively low interest sensitivity of money demand and the relatively small funds rate movements, the reentry path for money is essentially driven by the lagged dependent variable in money demand, regardless of the policy. Policies are mainly distinguished

by their effect on the paths for the funds rate and borrowing.

Monthly average reserve targeting with steady borrowing has four noteworthy features. First, like the discount window progressive pressure policy, this policy causes the funds rate to cumulate so that the funds rate peak in response to this money demand shock is put off until three weeks after the shock. Second, the weeks on either side of the peak funds rate week have a funds rate about 20 basis points below the peak. Thus this policy produces a relatively large temporary funds rate movement. Third, maintaining steady borrowing with monthly average reserve targeting alone produces an actual reentry rate in the .86 range, which is faster than the apparent actual desired rate .92, as calibrated in Section IV.[18] The relative restrictiveness of this policy stems from the fact that it targets nonborrowed reserves at its monthly average steady state value. Fourth, the policy, which is designed to smooth aggregate discount window borrowing in order to smooth the funds rate in fact does neither.

D. Pure Weekly Money Stock Targeting

This subsection abstracts from the monthly average aspects of money and nonborrowed reserve targeting in order to isolate the effect. of the gradual reentry component of the nonborrowed reserve operating procedure at a weekly level. For this discussion discount window administration does not include progressive pressure, so $(\Sigma \phi_i) B_t = f_t - d$. Technically, we assume that policy is implemented as

$$\overline{NBR_t} = NBR_t^*$$

(35)

$$NBR_t = \overline{NBR_t}.$$

In this case, policy amounts to providing week t nonborrowed reserves so that the expected week t money stock equals λM_{t-1}. Of course, if unanticipated shocks occur, realized money need not equal its weekly target.

The key to this policy's effect on the money stock is, of course, the size of λ, the reentry parameter. The policy's effect on the funds rate path depends on the size of λ relative to δ, the coefficient on the lagged dependent variable in money demand. As in the other cases, the one-billion-dollar week zero shock to money demand affects only the money stock in that

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[18] Ten weeks after the shock the simulated money stock is $(.86)^{10} M_0$.

week. The following week, when the aggregate money stock increase becomes known, the Federal Reserve adjusts NBR_i to yield $M_i = \lambda M_o$. Substituting this targeting expression into the money demand equation (16) and solving for the funds rate term yields

$$(36) \quad a_1 \sum_{i=0}^{3} b^i f_i = (\lambda - \delta) M_0$$

where $M_{i+1} = \lambda^i M_o$ and $M_o = 1$. Equation (36) immediately shows that if the Federal Reserve desires to choose a reentry rate λ so that the funds rate remains at its steady state value during the entire adjustment to a shock to money demand, it should choose λ equal to δ. In such a case, actual money will return to its steady state at a rate equal to the public's speed of adjustment δ.

Suppose the Federal Reserve's view of the speed of adjustment in money demand, δ_T, is incorrect, i.e., $\delta_T \neq \delta$. In this case, as long as the Federal Reserve chooses $\lambda = \delta_T$ it does not matter if $\delta_T \neq \delta$. Total reserve demand is approximately predetermined in each week t and the Federal Reserve can choose NBR and B to keep the funds rate at its steady state value. In this case, however, the money stock would converge back to the steady state after a disturbance at rate δ, not λ.

To isolate the effect of the gradual reentry rate λ without monthly average targeting in this model as calibrated, consider the results for A and $\delta_T = \delta$ values as given in Section IV. Since this case puts A slightly below δ, it has the Federal Reserve pulling the money stock back to its steady state value a bit faster than the actual speed of adjustment in the demand for money. Consequently, when the aggregate money stock becomes known in week one, the policy pushes the funds rate up to a peak 19 basis points above its steady state value. Because desired convergence on both the supply and demand side are both first order autoregressive, the funds rate, money, and borrowing, all converge monotonically back to the steady state. The money stock actually converges back to the steady state at rate $\lambda = .92$ because we have excluded monthly average targeting. In week ten money is approximately .44 billion dollars above its initial value and the funds rate is approximately 9 basis points above its steady state value. Notably, gradual reentry on a weekly basis alone is much more accommodative than either the discount window with progressive pressure or the monthly average targeting components of policy alone, which both put money at .21 billion dollars above its steady state in week ten.

VI.
SOME ANALYSIS OF THE COMPLETE NONBORROWED RESERVE OPERATING PROCEDURE

This section discusses the nonborrowed reserve operating procedure as a whole. We investigate the simultaneous effects of the four components of the policy : progressive pressure at the discount window; monthly average reserve targeting, steady borrowing, and money targeting at a gradual reentry rate. Within this context, each of the following subsections focuses on a particular characterization of the Federal Reserve's view of borrowing behavior, respectively : (A) a view that makes aggregate borrowing 'behave as a random walk, i.e., $B_t = B_{t-1}$, (B) a view that borrowing is only sensitive to the contemporaneous spread between the funds rate and the discount rate, i.e., $(\Sigma \phi_i) B_t = f_t - d$, and (C) a view in accord with the actual behavior of borrowing in the money market model, i.e., $\sum_{i=-3}^{3} \phi_i B_{t+i} = f_t - d$.

A. Random Walk Borrowing Behavior in the Federal Reserve's Monthly Average NBR Target Generating Model

A major difficulty in designing and implementing the NBR operating procedure was the choice of the initial borrowing objective. The Desk uses the initial borrowing objective to construct the monthly average nonborrowed reserve path. However, hard empirical knowledge of the behavior of aggregate discount window borrowing demand is difficult to obtain. In particular, the effect of an initial borrowing objective on the money stock depends on the relation between borrowing volume and the spread between the funds rate and the discount rate, a relation that is poorly understood. It is known that borrowing volume tends to be positively associated with the spread, but the size of that borrowing-spread relation seems difficult to pin down precisely.

In an effort to avoid having to employ a guess of this sensitivity when constructing the monthly average nonborrowed reserve path, the Federal Reserve originally assumed an initial borrowing objective near that prevailing in the most recent week.[19] We ap-

[19] See Federal Reserve Bank of New York (Summer 1980, p. 60).

proximate this method of choosing the initial borrowing objective by making borrowing a random walk in the Federal Reserve's monthly average target generating model.

However, our analysis of the NBR operating procedure shows that in this particular case the policy generates an unstable path.[20] Technically, the analysis establishes that this version of the NBR operating procedure is unable to return the money stock and the other variables to their steady state values after a disturbance. The analysis shows that after about six weeks of relative calm following a one-billion-dollar money demand shock, the policy generates explosive oscillations in money, the funds rate, and borrowing. Figure 1 illustrates these explosive responses.

This instability can be partially understood as follows. Suppose the desired money stock target is M_t^*. Hitting M_t^* requires producing a funds rate,

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[20] Research, undertaken after this paper was completed, has demonstrated that this specific instability is not robust to plausible changes in other elements of the model. For example, Moore, Porter, and Anderson (1985) analyses a closely related model in which the imposition of a random walk initial borrowing assumption is not in the least destabilizing. A key difference between their model and the present one is that the Desk is placed at the beginning of a six-week month rather than in the middle of a three-week month. Their model also captures the apparent reduced form properties of borrowing better than the present model even under the assumption that initial borrowing is interest sensitive. Thus, complete reckoning of the sources of stability and instability in the nonborrowed reserve targeting period awaits further investigation.

Figure 1
IMPULSE RESPONSES: RANDOM WALK INITIAL BORROWING ASSUMPTION
(1 Billion Dollar Money Demand Shock in Week 0)
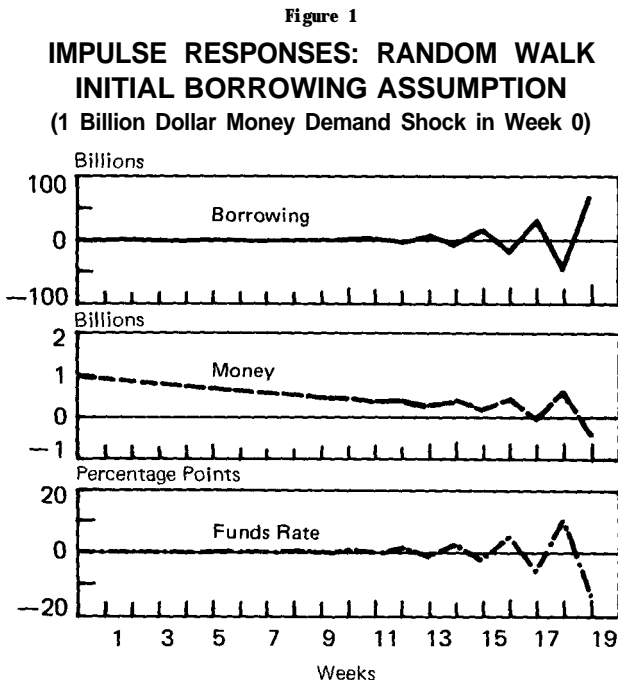
f_t^*, expected to induce the public to demand that quantity of money. Achieving f_t^*, in turn, requires using a guess of the interest sensitivity of borrowing to choose NBR_t in order to force a volume of borrowing B_t^* that will generate f_t^*.

Now suppose that the Federal Reserve sets the initial borrowing objective B_t equal to realized borrowing in the previous period. In general B_t^* will not equal B_{t-1}, and so this random walk borrowing objective will not achieve M_t^*. What's more damaging from a stability point of view is that by making borrowing behave as a random walk, the Federal Reserve introduces explosive swings into the funds rate and the money stock path. In practice the Desk, observing these large swings, would readjust its NBR path. But with random walk borrowing, the money stock has no tendency to return to its predisturbance steady state level.

The important point is that even though automatic funds rate increases associated with unexpected increases in money and reserve demand under nonborrowed reserve targeting might provide good protection against unexpected bursts of money growth, as long as policy tends to induce a random walk in borrowing it will nevertheless tend to induce a random walk in money.

B. Contemporaneous Borrowing-Spread Sensitivity in the Federal Reserve's Monthly Average NBR Target Generating Model

Apparently, the Federal Reserve came to realize the difficulties inherent in trying to keep the money stock in the neighborhood of its target while constructing the reserve path using a random walk borrowing objective. Consequently, the Federal Reserve replaced the random walk initial borrowing objective with an explicit rule of thumb relating borrowing to the spread between the funds rate and the discount rate. In other words, it appears that the Fed eventually began constructing its monthly average nonborrowed reserve target as described in Section II. B.

In any case, our model suggests that making this seemingly simple procedural change produces strikingly different results. As calibrated in Section IV, the model solution moves from being explosive to stable and generally well-behaved. In particular, the NBR operating procedure now succeeds in restoring the money stock, the funds rate, and borrowing back to their respective steady state values after any disturbance.

It is informative to look at the response of this complete nonborrowed reserve operating procedure to a one-billion-dollar money demand shock. Comparisons of the relevant responses between this policy and its individual components are shown in Figures 2, 3, and 4.[21] The complete NBR operating procedure has the funds rate rising by 14 basis points in week one and peaking at 15 basis points in the third week following the money demand shock. The money stock in the tenth week is .44 billion dollars above its steady state value; and the actual reentry rate is .92. The important point about the response pattern for the complete NBR operating procedure is that it closely approximates the response of pure weekly money stock targeting. In other words, the gradual reentry rate effect overwhelmingly dominates the progressive pressure discount window policy and the steady borrowing-monthly average components of the NBR operating procedure.

C. Progressive Pressure Borrowing Behavior in the Federal Reserve's Monthly Average NBR Target Generating Model

As it happened, at no time did the Federal Reserve document using leads or lags in its view of borrowing

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[21] Figure 2 shows only the money stock responses for two policies, progressive discount window pressure and the complete NBR procedure. In fact, the money stock path for the steady borrowing case virtually coincides with that for progressive pressure, and weekly money targeting almost duplicates the money stock path using the complete NBR procedure.

Figure 3

MONEY STOCK RESPONSE TO MONEY DEMAND SHOCK
(1 Billion Dollar Shock in Week 0)
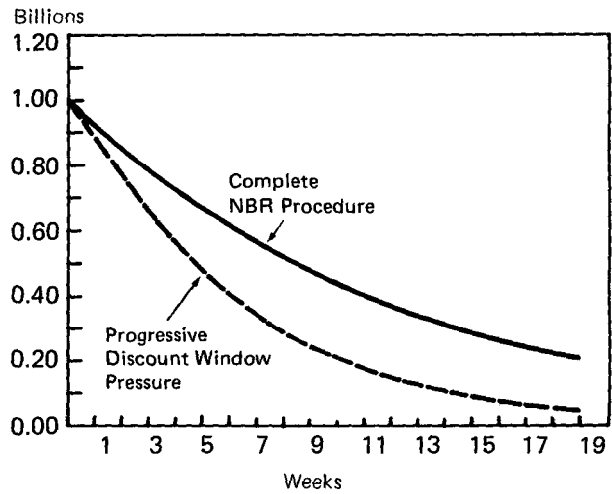
demand behavior in constructing its NBR targets. While the Federal Reserve did recognize that progressive pressure discount window administration would make borrowing demand depend on past realized borrowing and expected future borrowing, it had no reliable empirical estimates of the way leads and lags affect borrowing demand.

However, within our model it is possible to determine how policy would have been different if the Federal Reserve had used the model's true dynamic

Figure 2

FUNDS RATE RESPONSE TO MONEY DEMAND SHOCK
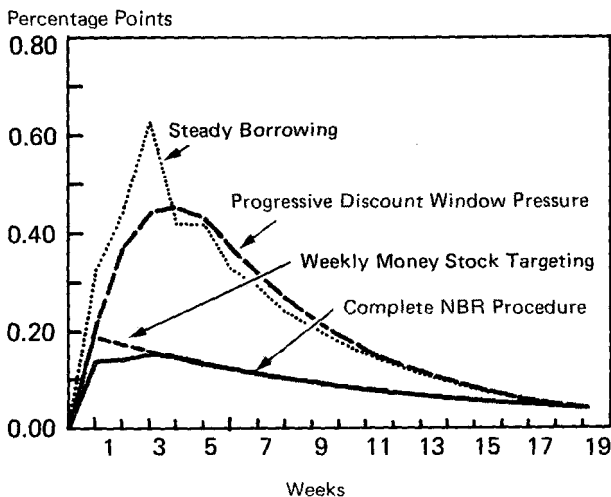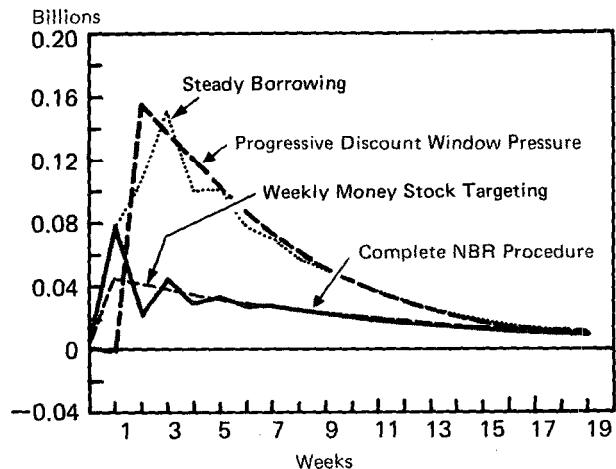**(1 Billion Dollar Shock in Week 0)**

Figure 4

RESPONSE OF ADJUSTMENT BORROWING TO MONEY DEMAND SHOCK
(1 Billion Dollar Shock in Week 0)

borrowing equation (4a) in its targeting procedure. In this case, using the parameter values given in Section IV, the model solution is still stable and well-behaved. The NBR operating procedure succeeds in restoring the money stock, the funds rate, and borrowing back to their respective steady state values after any disturbance. The response to a one-billion-dollar money demand shock is quite similar to the response described in Section VI. B. Here, the funds rate rises by about 16 basis points in week one and rises about 2 more basis points to its peak in week two. The money stock in week ten is .48 billion dollars above its steady state, reentering at a .92 rate. The major difference between these policies appears to be in the borrowing response. Here, borrowing in weeks one through four is respectively .08, .05, .03, and .03 billion dollars above its steady state level. For the case described in Section VI. B, borrowing in corresponding weeks is .08, .02, .05, and .03 respectively. In other words, borrowing moves more smoothly back to the steady state when the Federal Reserve has the correct view of borrowing in its NBR target generating model.

VII.
A SUMMARY OF RESULTS

The paper presents a theoretical analysis of the nonborrowed reserve operating procedure by decomposing it into four component parts: progressive pressure discount window administration, monthly average nonborrowed reserve targeting, steady borrowing, and pure weekly money stock targeting with a gradual reentry rate.

Progressive pressure at the discount window with fixed weekly nonborrowed reserves produces a cumulation in the funds rate in response to a money demand shock. Our specification of the lag length in the borrowing equation in conjunction with lagged reserve requirements causes the funds rate to peak five weeks after the shock occurs.

Given the reluctance to target on weekly money stock numbers, with their high noise to signal ratios, it was natural to introduce monthly averaging into the nonborrowed reserve targeting procedure. However, monthly average nonborrowed reserve targeting has its own set of problems. A "rolling month" formulation propagates forever an initial weekly nonborrowed reserve condition with a three-week period. A calendar month formulation truncates this propagation, but forces the Federal Reserve to face up to a last-week-of-the-month problem which can be equally troublesome.

Targeting monthly average nonborrowed reserves at its steady state value while imposing a steady borrowing restriction produces an outcome somewhat like progressive pressure discount window administration with fixed weekly nonborrowed reserves. The degree of money stock control is similar and the funds rate cumulates in the same way following a money demand shock. However, the steady borrowing-monthly average targeting procedure has some distinctive features. First, the weeks on either side of the funds rate peak (initiated by a one-billion-dollar positive shock to money demand) have funds rates about 20 basis points below the peak. Thus, the policy produces a relatively large temporary movement in the funds rate. Second, the policy which is designed to smooth aggregate borrowing in order to smooth the funds rate path in fact does neither.

There is no funds rate cumulation following a money demand shock for pure weekly money stock targeting along a gradual reentry path. The funds rate peaks the week following the shock. In this case the Federal Reserve can produce a flat funds rate profile during the period of adjustment by simply choosing the reentry rate λ to equal its view of the speed of adjustment in money demand, δ_T. Even if its estimate of δ is wrong, the procedure still produces a flat funds rate path. Although reentry will occur at a rate of δ instead of λ.

When the nonborrowed reserve operating procedure was initially implemented, the difficulty that the Federal Reserve had in obtaining empirical estimates of the relation between aggregate discount window borrowing and the spread between the funds rate and the discount rate led it to approximately use a random walk borrowing objective as a means of constructing the monthly average nonborrowed reserve target. Our analysis shows that this method of choosing the initial borrowing objective causes the model as calibrated in Section IV to be unstable. That is, it shows this policy to be unable to restore the money stock to its predisturbance steady state value after a disturbance.

Later in its experience with the nonborrowed reserve operating procedure the Federal Reserve appears to have replaced the random walk initial borrowing objective with an explicit rule-of-thumb relating borrowing to the funds rate-discount rate spread. The result of this seemingly simple procedural change in our model is striking. The model solution moves from being explosively unstable to being stable and well-behaved. In terms of its component parts, the response of this complete nonborrowed reserve operating procedure to a money

demand shock is surprisingly similar to the response of the pure weekly reentry money stock targeting component of policy. The contributions of progressive pressure at the discount window and steady borrowing with monthly average targeting are not readily apparent in the response of the overall policy.

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