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The Effects of Fiscal Policy in a Neoclassical Growth Model

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Abstract: This paper studies the effects of fiscal policies--depicted as stochasticchanges in government spending and distortionary tax rates--when the government is constrained from using lump sum taxes for achieving intertemporal budget balance. The ratio of debt to gnp, therefore, has consequences for the future choices of government spending and distortionary taxation and hence affects real economic activity. Further modeling fiscal policy in thisway generates results that differ substantially from those in standard stochastic models where lump sum taxes are used for budget balance. The modeling of fiscal policy presented here is also consistent with empirical evidence on U.S. fiscal policy.

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1. **Introduction**

This paper studies the effects of fiscal policies--depicted as stochastic changes in government spending and distortionary tax rates--when the government is constrained from using lump sum taxes for achieving intertemporal budget balance. This framework contrasts the more standard analysis in which spending and taxes follow exogenous Markov process and where lump sum taxation is used to balance the government's budget. Although we also model tax rates and spending as following Markov processes, the transition probabilities of these processes depend on the ratio of government debt to gnp. The ratio of debt to gnp, therefore, will have consequences for the future choices of government spending and distortionary taxation and hence will affect real economic activity.

Our depiction of fiscal policy gives bite to the restriction imposed by intertemporal budget balance since debt can not be viewed as a residual of policy that his dealt with via lump sum means. The results generated in our model can differ substantially from those in standard stochastic models. For example, the effects due to changes in the tax rate on capital depend on both the debt to gnp ratio and the persistence in the tax process. Even for processes that are fairly persistent, increases in the tax rate on capital can lead to increases in investment and this counterintuitive result is more likely to happen at very high or very low levels of the debt to gnp ratio.

Also, the economic effects of changes in government debt depend on the way that intertemporal budget balance is attained. If budget balance is primarily due to future changes in the tax rate on capital then debt crowds out investment. But unlike a standard Keynesian model higher debt ratios are associated with lower real interest rates. If on the other hand budget balance results from changes in the path of tax rates on labor, then investment is actually crowded in. It is only when government spending varies and taxes are held fixed that crowding out and higher interest rates are associated with higher ratios of debt.

Our model of fiscal policy implies that the debt to gnp ratio is mean reverting, which is consistent with evidence in Kremers (1989), King (1990), and Bohn (1991b). The model, despite its simplicity, also generates debt behavior that is reasonably consistent with U.S. data. The final section of the paper also indicates that our depiction of fiscal policy may help real business cycle models resolve some labor market anomalies.

2. **The Model**

The basic model is a standard neoclassical growth model into which we introduce distortionary taxation and government spending. These variables are modeled as Markov processes. To maintain intertemporal government budget balance the transition probabilities are functions of the debt to gnp ratio. As in Dotsey (1993) the stochastic process characterizing fiscal policy is endogenous and the government debt is mean reverting. Empirically, neither Kremers (1989) nor King (1990) can reject mean reversion in U.S. government debt, and Bohn (1991b) finds evidence that debt levels are mean reverting. Bohn (1991a) also shows that deficits are eliminated both by reductions in spending and increases in tax rates. Our model is consistent with these observations. Because all but the stochastic part of the model is standard, we give only a brief description of the model.

Firms

Firms maximize profits, d_t, which are remitted to households, by producing output via a constant return to scale technology that employs both capital, k, and labor, n. Both factors are rented from individuals. Capital is always supplied inelastically while we consider both inelastic and elastic labor supply. Formally,

PF:

$$\max_{\{k_t, n_t\}} \quad d_t = f(k_t, n_t) - r_t k_t - w_t n_t$$

where r is the rental rate on capital and w is the real wage. The first order conditions equate each factor's marginal product with its rental rate.

Individuals

Individuals maximize lifetime utility which depends on both consumption and leisure. They are endowed with one unit of time each period and an initial stock of capital. Individuals make their labor-leisure, consumption, and investment-saving decisions taking as given wage rates and rental rates. They also purchase one period government debt at a price p_t. Each bond pays one unit of consumption in the succeeding period. Consumers observe the current state of fiscal policy summarized by beginning of period per capita government debt, B_t, current tax rates on capital and labor income, τ^k and τ^n, and the current level of government spending. They also know current aggregate economic magnitudes such as output, the capital stock, employment, investment, and end of period debt B_{t+1}. Formally, the individual's problem, PI, is written

PI:

$$\max_{\{c_t, n_t, b_{t+1}\ k_{t+1}\}} \quad U = E_o \sum_{t=o}^{\infty} \beta^t u(c_t, 1-n_t)$$

subject to

$$c_t + i_t + p_t b_{t+1} \leq (1-\tau_t^n) w_t n_t + (1-\tau_t^k) r_t k_t + b_t + TR_t$$

$$k_{t+1} = (1-\delta) k_t + i_t$$

where TR is aggregate per capita transfers, and lower case variables indicate

values at the individual level.

Maximization yields the following first order conditions

(1a) $u_2(c_t, 1-n_t) = u_1(c_t, 1-n_t)(1-\tau_t^n) w_t$

(1b) $u_1(c_t, 1-n_t) = \beta E_t\{[(1-\tau_{t+1}^k) r_{t+1} + (1-\delta)] u_1(c_{t+1}, 1-n_{t+1})\}$

(1c) $p_t u_1(c_t, 1-n_t) = \beta E_t u_1(c_{t+1}, 1-n_{t+1})$

where u_j refers to the partial derivative with respect to the j^{th} argument.

Fiscal Policy

The government spends resources and finances its spending through taxes and debt. Debt evolves according to

$$(2) \quad p_t B_{t+1} = G_t + B_t - \tau_t^k r_t K_t - \tau_t^n w_t N_t + TR_t$$

where capital letters refer to per capita aggregate quantities. G is government spending, B is the stock of one-period bonds outstanding, and TR is the level of transfers. Tax rates on capital and labor income, τ^k and τ^n, and the ratio of government spending to gnp, $\tilde{g}$, depend on the debt to gnp ratio, $\tilde{b}$.[1] Government budget balance is achieved through changes in distortionary taxation and government spending. Specifically, we model the elements of fiscal policy as a two-state Markov process with transition probabilities given by

$$(3a) \quad \text{prob}\ (\tau_{t+1} = \tau_\ell \mid \tau_t = \tau_\ell) = \min\ \{\max[(1-\gamma\tilde{b}_t)^{1/\mu}, 0], 1\}$$

$$(3b) \quad \text{prob}\ (\tau_{t+1} = \tau_h \mid \tau_t = \tau_h) = \max\ \{\min[\gamma\tilde{b}_t^{1/\mu}, 1], 0\}$$

[1]We focus on the ratio of government spending to gnp rather than the level of spending because the ratio is stationary making it easy to extend our analysis to economies with steady state growth. One could easily add growth to our model by including technical progress in labor productivity. In that case one could interpret our model as represening deviations from trend as in King, Plosser, and Rebelo (1988).

(4a) prob $(\tilde{g}_{t+1} = \tilde{g}_\ell \mid \tilde{g}_t = \tilde{g}_\ell) = \max\{\min[\gamma\tilde{b}_t^{1/\eta}, 1], 0\}$

(4b) prob $(\tilde{g}_{t+1} = \tilde{g}_h \mid \tilde{g}_t = \tilde{g}_h) = \min\{\max[(1-\gamma\tilde{b}_t)^{1/\eta}, 0], 1\}$

where the subscripts ℓ, h refer to low and high values respectively. These transition probabilities imply that the debt to gnp ratio is bounded and only rarely lies outside the interval $[0, 1/\gamma]$. As $\tilde{b}$ approaches a value of $1/\gamma$, taxes will be high and spending will be low with probability one. As long as a combination of high taxes and low spending reduces debt, the debt to gnp ratio will be driven down. Similarly as $\tilde{b}$ approaches zero the economy will be in a low-tax, high-government-spending state and the debt will rise. Thus, there is some tendency for debt to revert toward its mean.[2] In what follows we will call this policy a managed debt policy.

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[2]The debt to gnp ratio can temporarily move outside $[0, 1/\gamma]$ because next period's taxes and spending depend on this period's debt to gnp ratio. For example, the current state could be $\tau_t = \tau_\ell$, $\tilde{g}_t = \tilde{g}_h$, $\tilde{b}_t = (1/\gamma) - \varepsilon$. Given this state it is possible that next period's taxes and spending will not change. Thus tomorrow's debt/gnp could exceed $1/\gamma$ and the debt/gnp two periods hence could be larger still. However, since $\tilde{b}_{t+1} > 1/\gamma$ implies $\tau_{t+2} = \tau_h$ and $\tilde{g}_{t+2} = \tilde{g}_\ell$ the debt to gnp ratio will start to decline. Since a combination of τ_ℓ, $\tilde{g}_h$ can only increase $\tilde{b}$ by so much, $\tilde{b}$ is bounded above. Similarly, $\tilde{b}$ is bounded below. Further our process for fiscal policy rules out any Ponzi games. That is $\lim_{T\to\infty} E_t[P_T B_{T+1} / \prod_{s=t}^{T} (1/P_s)] = 0$ for equilibrium paths in this model.

The parameters μ and η control the persistence of the tax and spending processes. As these parameters increase the probabilities of remaining in a given tax or spending state increase for any value of the debt to gnp ratio.

Equilibrium

Equilibrium is a set of functions representing quantities and prices that solve the firms and consumers maximization problems, do not let either consumers or the government borrow more than can be repaid, and obey the following aggregate equilibrium conditions.

(5) $C_t + I_t + G_t = f(K_t, N_t)$

(6) $b_t = B_t$

(7) $k_t = K_t$

(8) $n_t = N_t$

We solve for equilibrium by first using equation (5) to substitute out consumption. Equation (1a) together with the relationship $w_t = f_2[K_t, N_t]$, and equations (7) and (8) are then used to solve for labor $n_t = n(k_t, \tilde{b}_t, \tau^n_t, \tau^k_t, \tilde{g}_t, k_{t+1}) = n(s_t, k_{t+1})$ where the state $s_t = (k_t, \tilde{b}_t, \tau^n_t, \tau^k_t, \tilde{g}_t)$. We then substitute for labor in equation (1b) to yield

(9) $u_1[f(k_t, n(s_t, k_{t+1})) + (1-\delta)k_t - g_t - k_{t+1}, 1-n(s_t, k_{t+1})]$

$$= \beta E_t[(1-\tau^k_{t+1})f_1(k_{t+1},n(s_{t+1},k_{t+2})) + (1-\delta)]$$

$$\times u_1[f(k_{t+1}, n(s_{t+1}, k_{t+2})) + (1-\delta)k_{t+1} - g_{t+1} - k_{t+2}, 1-n(s_{t+1}, k_{t+2})]$$

Equation (9) is a nonlinear second order stochastic difference equation. Given n(s, k') where the "'" indicates next period's value of a variable, we solve for the function, k' = h(s) which is the fixed point of (9). This equilibrium policy function for k' then yields the equilibrium policy function for labor n, because n was a function of arbitrary k'. At each step of the iteration we use equations (1c) and (2) to determine $\tilde{b}'$ based on the current state s and the policy functions n and h. The algorithm is similar to the discrete state space method described in Baxter (1991) and Dotsey and Mao (1992).

3. **Stochastic Taxes**

We can highlight the effects of distortionary taxation by comparing an equilibrium generated by a policy with managed debt with the standard case in which taxes follow an exogenous Markov process. Our comparisons are based on an examination of policy functions, impulse response functions, and impact effects. To understand the effects of fiscal policy, we proceed sequentially by first taking the simplest case--a stochastic tax rate on capital and a fixed tax on labor with inelastic labor supply--and then proceed to the more general cases.

The experiments in this section are dynamic stochastic analogs to comparative static analysis. Our fundamental concern is understanding the workings of a fairly intricate fiscal policy process. We use post-Korean War U.S. data as a rough guide for calibrating the models. We fix the ratio of government spending to gnp at .18, which is the ratio reported in Christiano and Eichenbaum (1991). We also fix the level of transfers at 8% of gnp. In our experiments the debt to gnp ratio essentially lies between 0 and 1/2. Until recently, measured government debt/gnp has remained within this range. Picking a limited range also helps conserve on grid points.

Our remaining parameter values are within the realm of most real business cycle models. Labor's share of gnp is chosen to be .6, utility is logarithmic and separable in consumption and leisure, the discount factor is .97, and the depreciation rate on capital is .10. We parameterize the utility function so that individuals spend 20% of their time working.[3]

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[3]This number is taken from King, Plosser, and Rebelo (1988).

(a) Fixed Labor Supply with the variable tax rates on income from capital

In this example we allow the tax rate on capital to vary and use a persistance parameter of μ=4. With this parameter, tax rates are unlikely to change for most of the values for the debt/gnp ratio. [4] The tax rate on capital takes on the value of either .20 or .50. The mean of the tax rate is .37 with a standard deviation .149 and an AR1 coefficient of .64. This parameterization is roughly consistent with one of the series reported in Auerbach and Hines (1988) which has a mean of .40, a standard deviation of .141, and an AR1 coefficient of .82. We choose a somewhat lower than actual persistence to illustrate an interesting result, that it can be optimal for agents to invest more when taxes are high even when taxes on capital are persistent.

The policy functions for capital and consumption, and the equilibrium function for the real after-tax rate of interest are displayed in Figure 1. The policy functions are drawn for a capital value chosen from the middle of capital's ergodic set. As shown, the capital stock in the high tax

[4] For example, the probabilities of taxes remaining in the low-tax state for debt/gnp ratios of (-.10, -.063, -.026, .011, .047, .084, .121, .158, .195, .232, .268, .305, .342, .379, .416, .453, .489, .526, .563, .60) are (1.0, 1.0, 1.0, .99, .98, .95, .93, .91, .88, .85, .82, .78, .75, .70, .64, .55, .38, 0, 0, 0). It is not until the debt/gnp ratio reaches .49 that next period's tax rate is more likely to be high than low.

state (dotted line) lies above the capital stock in the low tax state. This result implies that investment is higher when taxes are high even though a high tax rate today generally implies a high tax rate next period. This result is the same as the one in Dotsey (1993) for an economy using a linear technology and occurs for the same reason. A high tax rate today lowers the debt to gnp ratio implying that the future path of taxes will be lower and that investment is profitable. This response is only optimal if tax rates are not too persistent. If we set μ=6 implying an AR1 coefficient on taxes of .69, agents will invest less when taxes are high. Therefore, for a tax process displaying persistence that conforms more closely to the data investment will fall when the tax rate rises. Further, investment declines with debt because higher debt levels implies higher future taxes.

The above result stands in sharp contrast to the standard tax literature [5], where labor supply is typically fixed and taxes follow a Markov process. As long as tax rates are positively correlated the standard case implies that high taxes today result in higher future tax rates reducing investment.

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[5]For example see Coleman (1991) or Dotsey (1990). In a nonstochastic environment see Judd (1987), Abel (1982), Abel and Blanchard (1983), Becker (1985), Brock and Turnousky (1981), Danthine and Donaldson (1985), and Hall (1981).

The policy function for consumption is a mirror image of the policy function for capital. With inelastic labor supply investing more implies consuming less. The equilibrium function for interest rates is also shown in Figure 1 and its shape is related to the policy function for consumption. Interest rates are lower in the high tax state due to the upward slope of the consumption policy function. When taxes are high today, debt and consumption will fall next period, while if taxes are low, debt and consumption will rise. This implies that for any given debt level interest rates in the high tax state lie below those in the low tax state. The interest rate equilibrium functions are also downward sloping attaining their lowest value when debt is high. In the high tax-high debt state there is little probability that a low tax rate will occur tomorrow, hence the expected consumption decline is relatively large implying a low real interest rate. In the low tax state there is a reasonably high probability that high taxes will occur tomorrow, implying a relatively small expected increase in consumption and hence a lower real interest rate. Similarly rates are higher when the debt is low.

The extent to which debt is non-neutral in our model can be illustrated by the elasticity of the various policy functions with respect to debt around the steady state debt to gnp ratio (see Table 1) and by the correlations between debt and other endogous variables (see Table 2). An increase in debt crowds out investment and slightly increases consumption. The non-neutrality in this model differs from a standard Keynesian model because real rates in this model are negatively related to the level of debt. These features also appear in the correlation coefficients which show a negative correlation between debt and investment as well as a negative correlation between debt and the real interest rate.

(b) Variable labor supply with variable tax rates on income from capital

For these experiments we keep the same parameter values but allow labor to vary. The policy functions for capital, labor, consumption, and the equilibrium function for the real after-tax interest rate are depicted in Figure 2. The policy functions for capital and consumption differ from those in the fixed labor case. With varying labor, agents now invest more, work more, and consume less in the low tax state over much of the debt space.

Variable labor creates another degree of freedom in the model. With labor fixed, changes in investment must be offset one for one with changes in consumption. With variable labor that need not be the case since output can adjust contemporaneously. Variable labor allows consumption to be much smoother and at the same time allows investors to take advantage of low persistent marginal tax rates.

Persistence of the tax processes also plays a role in the shape of the policy functions. Reducing the persistence of the tax series by setting μ=2.5, which implies ρ=.53 yields the same qualitative results as the fixed labor case. Crossovers in the policy functions occur because the expected duration of remaining in any particular state depends on the value of the debt to gnp ratio. For example, if debt were high and taxes were low, agents would expect taxes to rise and stay high for a greater number of periods than if taxes were currently high. Hence they invest less in the low tax state. One surprise is that variable labor has little affect on the equilibrium function for real interest rates. The interest rate depends on intertemporal rates of substitution and, therefore, depends on next period's consumption. The consumption policy function is drawn for a specific value of capital, and capital is changing over time. Because next period's capital is higher in the low-tax state next period's consumption will be higher despite rising debt. The shift in the curves due to capital accumulation dominates movement along the curve and there is greater consumption growth when taxes are low.

Evaluating the elasticities of the various policy functions with respect to debt and the correlation coefficients leads to the conclusion that only half of the standard Keynesian story occurs. Higher debt crowds out investment but reduces the interest rate.

(c) Variable labor with a varying labor tax and fixed tax on capital income

We next examine the effects of varying the tax on labor income rather than the tax on capital. Here we allow labor tax rates to vary between .16 and .24. With $\mu=8$, these rates have a mean of .217, a standard deviation of .036, and an AR1 coefficient of .76. Using post-Korean War data our tax process matches the one constructed by Barro and Sahasakul (1986), which has a mean of .278, a standard deviation of .039, and an AR1 coefficient for their detrended series of .78.

Intratemporal substitution effects in the labor-leisure decision dominate the results. Individuals substitute labor effort into low tax states, driving up the marginal productivity of capital and hence increasing investment demand. Greater labor effort results in more output and more is invested. As debt rises, the probability of high taxes next period increases thus inducing individuals to take even greater advantage of the current low tax rate. In the low tax state, high debt means that future taxes are more likely to be high so the incentive to work is greater than when debt is low. Thus the policy function for labor effort is upward sloping (see Figure 3).

Because the policy function for both labor and capital are now upward sloping (a non-Keynesian result) the policy function for consumption is downward sloping even though there is more output available at high levels of debt. Agents, however, consume and invest more in the low tax state due to increased labor effort and greater output. As in the previous case interest rates are higher when taxes are low. This is because capital and, therefore, next period's consumption increase when taxes are low.

The variable tax on labor income creates crowding in rather than crowding out, just the opposite of the standard Keynesian story. The policy function for investment has a positive elasticity and positive correlation with respect to debt while the real interest rate is negatively correlated with debt.

The managed debt case also yields somewhat greater impact effects than the standard exogenous Markov case because of the stronger intertemporal substitution effects on labor effort (see Table 3). With debt management, lower current taxes imply a higher future path of taxes making agents work even harder today. The greater impact in effort feeds over into output and investment.

(d) Taxing both labor and capital

In this example both labor and capital are taxed at the same rate, which is equivalent to a production tax. The results are a hybrid of the results in the last two sections. The large divergence in policy functions (Figure 4) between high and low tax states reflects the responsiveness of labor to a tax on wage income. The negative slope of the capital and labor policy functions as well as the positive slope of the consumption policy function reflect the influence of the tax on capital. Because this case is hybrid of the previous two experiments, the elasticity of investment with respect to debt is greatly diminished from the case when only τ^k varies. Thus with a production tax there is much less crowding out than in the case where only income from capital is taxed. The interest rate, however, varies indirectly with government debt and thus only half of the traditional Keynesian story holds.

4. Government Spending

This section examines the effects of government spending. To highlight the differences from standard models, we first keep tax rates constant throughout and allow lump sum taxes to balance the budget when spending follows an exogenous two state Markov process. When there are no lump sum taxes government spending must adjust so that the debt to gnp ratio is bounded. We allow government spending relative to gnp to vary between .14 and .22. Its mean is .17 in the following experiments and its standard deviation is .039. The parameter η is varied between 6 and 1 implying AR1 coefficients of .73 and .10. This allows us to explore the effects that persistence has on economic activity. The government taxes production at the constant rate of 26%. After isolating the effects of government spending, we allow tax rates and spending to vary simultaneously.

(a) Persistent Government Spending

We assume that government spending is useless. The economic response to changes in government spending, therefore, mainly arise through wealth and crowding out effects. The policy functions in Figure 5, show that agents work harder and consume less when spending is high. Although high government spending causes high output through increased labor effort, output rises by less than government spending. Hence next period's capital stock falls.

As debt rises the expected future path of government spending falls. The policy function for labor is, therefore, downward sloping with respect to debt while the consumption policy function is upward sloping. As labor hours decrease, output and the capital stock fall. Hence debt crowds out investment. High government spending raises interest rates motivating agents to work harder and consume less. As the debt rises, implying less future government spending, labor effort, capital, and consumption growth decline. Thus the equilibrium function for interest rates is downward sloping with respect to debt.

Even though the equilibrium function for the interest rate is negatively related to debt, the correlation between interest rates and debt is positive. The intuition can be seen by examining the economy's response to a high government spending shock, which is displayed in Figure 6. Debt rises when spending is above its average value causing spending to eventually fall below its steady state expected value. This mild oscillatory behavior in spending sets up oscillatory behavior in the other variables. As spending falls and debt rises, labor effort declines. However, declining government spending allows agents to increase consumption and investment even though output mimics the behavior of labor. The real rate is generally above its steady state value as a result of consumption growth, so the correlations between debt and investment and debt and interest rates resemble the predictions of standard Keynesian models. Investment is below average when the debt is relatively high while interest rates are above average.

With the exception of labor (and as a result output), the behavior of the other endogenous variables is not strikingly different from what occurs when spending follows an exogenous Markov process. The impact effects in Table 4 show that labor responds with more vigor to an increase in government spending when spending follows a Markov process. In the debt management case higher spending raises the level of debt implying that future spending must be lower than it otherwise would have been. The wealth effects are, therefore, smaller than when spending is exogenous. [6]

(b) The effects of lowering persistence

When the persistence in government spending is greatly reduced by setting $\eta=1$ implying an AR1 coefficient on spending of .10, the results for the exogenous Markov process and the managed debt process are very similar (see Figure 7). Government spending is more transitory and causes smaller wealth effects. Thus the impact effects of a rise in spending are much smaller (see Table 4 and Figure 7). These results are consistent with those in Aiyagari, Christiano, and Eichenbaum (1991) and Baxter and King (1993). Also, because government spending changes states so frequently the debt doesn't fluctuate very much and the path of shocks generated

[6] We calculated the present value of government spending to be about 10% less for the mean reverting debt policy in this example.

by each process are almost identical. As a result all endogenous variables behave in a like manner.

(c) The effects of very high persistence

In this experiment we examine McGrattan's (1992) suggestion that very high persistence in government spending can lead to increased investment in the high spending state. To generate high persistence we set η=100 which corresponds to an AR1 coefficient of .92. We find that with log utility and hence a relative risk aversion parameter of σ=1 it is possible for investment to be higher when spending is high, but only over a narrow range of the debt space. With an exogenous Markov process for spending, investment is higher when spending is high, but this result is sensitive to the degree of relative risk aversion. With increased risk aversion (σ=2) investment is lower when spending is high in both the managed debt and exogenous Markov process cases.

The reason for the disparity in results is that with debt mangement the wealth effects of high or low government spending are almost identical near the boundaries of the debt space. If, for example, debt levels are very high the probability that next period's government spending will be low and stay low is high no matter what the current state. Therefore, labor effort and consumption do not differ by very much across spending states and the major difference across the two states is in investment. In particular, investment is lower in the high spending state. An analogous argument indicates that investment is lower in the high spending state when debt is very low. It is only in the middle of the debt space that the wealth effects of high spending can cause enough of an increase in labor effort and decline in consumption that investment is higher. The large increase in labor effort also increases the marginal product of capital reinforcing the wealth effects on consumption and investment. When government spending follows an exogenous Markov process the persistence of the process is independent of debt levels. Therefore, wealth effects and the accompanying substitution effects are either strong enough to encourage investment when spending is high or they are not.

An increased persistence in government spending and the accompanying higher investment in the high spending state results in greater consumption variability as well. With CRRA utility, an increase in relative risk aversion implies a reduction in the elasticity of intertemporal substitution of consumption. With agents less willing to substitute intertemporally, investment becomes less variable, and therefore it is less likely that investment will rise in response to high government spending.

(d) Taxes and Spending Both Vary

In this case we now add persistent taxes and compare how simultaneously varying taxes and spending affects behavior. These comparisons are done by examining the impulse response functions in figures 8 and 9 which are responses to a high spending-low tax shock and a high spending-high tax shock.

The combination of low taxes and high spending is more expansionary than just lowering taxes or increasing spending. The tax induced substitution effects augment the wealth effects of government spending implying that labor effort increases by a large amount. This increases output by enough so that the impact effect on both consumption and investment is positive.

When the initial impulse to taxes is high, (Figure 9) the impact effect of fiscal policy is reversed. With an increase in the tax rate substitution effects outweigh wealth effects and labor effort falls. The fall in labor effort results in lower output, consumption, investment, and a drop in the real rate of interest. Thus the expansionary effect on output of government spending programs can be totally overturned if they are financed out of current tax revenue. This latter result is consistent with the analysis in Baxter and King (1993).

5. **Implications for Debt Behavior and Business Cycles**

(a) Debt

In this section we parameterize our tax and government spending processes to roughly match the actual post-1916 stochastic processes exhibited in Bohn's (1991a) data on U.S. fiscal policy. [7] To do this requires some essential modifications both to the permissible debt space and the stochastic structure. The mean reverting debt model with two states generates too much oscillatory behavior. We thus construct a hybrid process that allows taxes and spending to follow exogenous Markov processes on some portion of the interior of the debt space but force both processes to be responsive to debt/gnp ratios near the boundaries. Specifically, we use three states for tax rates and two for government spending. The admissible range for the debt to gnp ratio is [-.1, 1.1]. The model generates tax data that has a mean of .14, a standard deviation of .04, and an AR1

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[7] We use his data because it doesn't net out any components of government spending. If we are to have any chance of matching the series on debt we must either use inclusive measures or model the different components of spending separately. We start in 1916 because that is the inception of income taxes, and the data over the entire sample, 1800-1988, does not appear to be generated by the simple model in this paper (i.e. the mean of government spending and tax revenue vary greatly over the last two centuries). To match the data we would need more than one fiscal policy regime. As it is the model is forced to confront two major wars in order to get enough data points for the spectra to have any meaning. What we would like is 100 years of post-Korean war data.

coefficient of .87, while government spending has a mean of .15, a standard deviation of .07, and an AR1 coefficient of .80. Our parameterization thus produces tax rates and spending ratios close to post-1916 data. The comparable statistics for the data are .14, .04, and .89 for taxes and .158, .08, and .80 for government spending. The spectra for actual debt to gnp ratios and the average of 200 simulations of the model are shown in Figure 10 along with the coherence between actual debt and the model's debt (one standard deviation error bands are represented by the dotted lines.)[8] The coherence is roughly 50 percent with a downward spike at approximately the frequency exhibited by wars. Given the simplicity of the model, its ability to match actual debt behavior this well is encouraging. For instance, the level of coherence is higher than that displayed by real business cycle models for many relevant economic magnitudes (see Watson (1990)).

[8]The spectra were estimated using linearly detrended data. Since the model data do not display any trend the model data is in deviation from mean form.

(b) business cycles

Recent work by Braun (1988), Christiano and Eichenbaum (1991) and McGrattan (1988, 1991) indicate that including fiscal policy in standard real business cycle models can produce noticeable improvements in the fit of these models, especially with respect to labor market behavior. Most RBC models understate the relative volatility between hours and output and overstate the relative volatility between productivity and hours. The models also overstate the correlation between labor productivity and output and labor productivity and hours (see Hanson (1985), King, Plosser and Rebelo (1988), and Christiano and Eichenbaum (1991)).

An increase in government spending produces negative wealth effects which induce more labor effort and more output. Because of labor's declining marginal product, government spending shocks reduce average productivity and set up a negative correlation between average product and either output or hours. This negative correlation, however, only occurs when government spending follows an exogenous Markov process (see column 7, Table 5). [9] In this case all increases in spending are financed by lump sum taxes and future spending does not have to respond to budget imbalance. This modeling

[9]For both the case of managed debt and the exogenous Markov process, g varies between .22 and .30. The standard deviations for g are .0387 and .04, respectively.

of fiscal policy is at odds with the empirical findings of Bohn (1991a) who finds that 65-70 percent of a deficit caused by higher spending is reduced by decreases in future spending. Thus a process that captures this type of behavior represents a more realistic model.

In the more realistic managed debt case, when only government spending varies, the relevant correlations are only somewhat lower than those produced by standard RBC models (see column 6, Table 4). The positive correlation between average productivity and either output or hours occurs despite the fact that labor hours and output rise on impact while wages fall. The impulse response functions in Figure 5 show that labor hours (or output) and wages are below their steady state values in periods 5-15 and above their steady state values in periods 15-25. Labor hours and the wage rate both reach their minimums when debt levels reach their maximum. As debt increases the expected future path of spending falls below its steady state value. At the same time the capital stock has fallen and wages remain below their steady state value. This behavior accounts for the positive correlation displayed by these two series.

When spending and taxes both vary, the debt management policy produces the desired negative correlations in the labor market and a lower relative variability in average productivity than a technology shock (see column 8,

Table 5).[10] In this case there is also little to distinguish the managed debt process from the exogenous Markov process, indicating that when taxes and government spending jointly depend on debt, our methodology should be able to replicate much of the improvement in RBC models reported by Braun (1988) and McGrattan (1988, 1991).

6. **Conclusion**

This paper has examined an alternative methodology for studying the effects of fiscal policy. Our model of fiscal policy takes the consequences of intertemporal budget balance seriously and at the same time allows for uncertainty in the fiscal policy process. The combination of these two elements is able to generate behavior that is, in some instances, strikingly different from standard results. Namely debt is non-neutral, the expansionary effects of government spending are dampened, and the taxation of capital can have

[10] In this experiment with $\mu = \eta = 10$, taxes and spending are highly persistent. Tax rates take on values of either .228 or .292 and g takes on values of either .24 or .28. We do this in order to lower the variability of both processes to better conform with the data. The standard deviation of tax rates is .03 which corresponds to the Barro and Sahasakul (1986) series while that of the ratio of government spending to gnp is .019. The latter figure is consistent with the standard deviation of spending net of military, transfers, and debt financing relative to gnp over 1947-1988. However, $\sigma_G/\sigma_y = 2.33$ (1.60 for the Markov case). This latter figure is still somewhat higher than the 1.15 figure reported by Christiano and Eichenbaum (1991). Additional sources of output variability would allow us to better match this data.

surprising and counterintuitive results. The model generates cases where debt crowds in investment and the behavior of the real interest rate differs from behavior portrayed in standard Keynesian models. The model is also consistent with empirical evidence on U.S. fiscal policy as well as with the behavior of U.S. government debt. Finally this modeling strategy shows promise in helping to correct some of the labor market anomalies found in standard real business cycle models.

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TABLE 1

Elasticity of Policy Functions Around Steady State Debt/GNP Ratios*

	Case 1		Case 2		Case 3		Case 4		Case 5	
	τ^k(low)	τ^k(high)	τ^k(low)	τ^k(high)	τ^n(low)	τ^n(high)	τ(low)	τ(high)	g(low)	g(high)
c	.079	.079	.058	.061	-.009	-.007	.015	.015	.024	.028
n	.000	.000	-.090	-.095	.014	.012	-.021	-.026	-.038	-.042
i	-.267	-.264	-.427	-.474	.058	.056	-.071	-.127	-.141	-.185
y	.000	.000	.004	.004	-.001	-.005	.001	.001	.001	.001
r	-.122	-1.14	-.399	-1.47	-.020	-.018	-.146	-.478	-.079	-.083

*Case 1 is for fixed labor and variable tax rates on rental income from capital; Case 2 is for variable tax rates on capital with variable labor; case 3 is for variable taxes on labor income; case 4 is for variable taxes on total income, and case 5 is for variable government spending.

TABLE 2

Correlation Coefficients with Respect to the Debt to GNP Ratio

(3000 observations)

Case	I	r	C	N	Y
τ_K varies N fixed (μ=4)	-.95	-.26	.75	na	-.24
τ_K varies N varies (μ=4)	-.93	-.40	.40	-.92	-.69
τ_N varies (μ=8)	.08	-.72	.45	-.03	.25
τ varies (μ=10)	-.15	-.54	.85	-.02	.22
g varies (η=6)	-.57	.93	-.72	-.51	-.80
g varies (η=1)	-.40	.99	-.96	-.37	-.74
g varies (η=6), τ varies (μ=4)	-.21	-.52	.68	.09	.19
g varies (η=10), τ varies (μ=10)	-.47	-.42	.51	-.20	-.08

TABLE 3

Impact Effects for a Decline in Taxes

(measured as minus the ratio of the percent deviation
from steady state values to the percent deviation
in the decline in taxes)

Case	Y	N	C	I	r	W
					Managed debt	
τ_K varies N fixed (μ=4)	0	0	.004		-.008	
	1.73	0				
τ_K varies N varies (μ=4)	.010	.011	-.006		.061	
	1.53	-.003				
τ_N varies (μ=8)	.194	.327	.062		.604	
	.508		.145			
τ varies (μ=10)	.296	.494	.056		.934	
	2.09	.175				
			Markov			
τ_K varies N fixed (ρ_{τ_K} =.64)	0	0	-.077		.24	1.41
	0					
τ_K varies N varies (ρ_{τ_K} =.64)	.057	.094	-.060		.421	
	1.48	-.036				
τ_N varies (ρ_{τ_N} =.76)	.171	.286	.063		.510	
	.385		.134			
τ varies (ρ_{τ}=.70)	.304	.504	.030		1.016	
	1.549		.155			

TABLE 4

Impact Effects for a Rise in Government Spending

Case	Y	N	C	I	r	W
			Managed debt			
τ fixed η=6	.049		.082		-.052	
	-.434		.221		-.032	
τ fixed η=1	.020		.033		-.021	
	-.625		.126		-.013	
τ varies η=6, μ=4	.407		.680		.062	
	.490		2.20	.223		
τ varies η=10, μ=10	.388		.653		-.017	
	.651		2.58	.144		
			Markov			
τ fixed ρ_g=.73	.083		.138		-.090	
	-.265		.325		-.055	
τ fixed ρ_g=.10	.038		.062		-.040	
	-.559		.187		-.025	
τ varies ρ_τ=.56, ρ_g=.60	.507		.844		-.029	
	1.027		2.10	.183		
τ varies ρ_τ=.77, ρ_g=.81	.445		.744		-.088	
	1.01	2.40	.097			

TABLE 5[1]

Correlation Coefficients and Relative Variances

U.S. Data   RBC Model   G varies(η=6, ρ_g=.73)   G varies(η=10, ρ_g=.81)

τ fixed τ varies (η=10, ρ_τ=.77)

Managed H CE   Managed H CE KPR

Debt Markov   Debt Markov

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
σ_c/σ_y	.73	.44	.31	.57	.64	.39	1.70	.67	.72
σ_i/σ_y	4.89		3.14		2.31	3.76	3.26	2.38	2.61
σ_n/σ_y	.94	.86	.52	.36	.48	.80	2.01	1.31	1.29
$\sigma_{y/n}/\sigma_y$	.67	.71	.50	.67	.69 [2]	.40	1.21	.52	.59
$\sigma_{y/n}/\sigma_n$	.71	.61	.97	1.85	1.43 [3]	.50	.60	.40	.46
corr (c,y)	.85		.89		.87	.19	-.75	.50	.58
corr (i,y)	.92		.99		.92	-.03	-.99	.77	.85
corr (n,y)	.76		.98		.79	.92	.90	.93	.90
corr (y/n,y)	.42		.98		.90 [4]	.64	-.67	-.42	-.27
corr (y/n,n)		.16		.95		+.29	-.93	-.72	-.66

[1]The data reported in this table are from Hansen (1985) and the establishment data in Christiano and Eichenbaum (1991). The moments for RBC models are from Hansen (1985) and

Christiano and Eichenbaum (1991) for the cases where labor is divisible and government spending has no value. The model moments from King, Plosser, and Rebelo (1988) and for the Long and Plosser model with realistic depreciation and persistence in the technology shock.

[2]This ratio is the one reported for wages and output.

[3]This ratio is the one reported for wages and hours.

[4]This correlation is the one reported for wages and output.