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The Optimal Rate of Inflation with Trending Relative Prices

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The Optimal Rate of Inflation with Trending Relative Prices*

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Abstract

The relative prices of different categories of consumption goods have been trending over time. Assuming they are exogenous with respect to monetary policy, these trends imply that monetary policy cannot stabilize the prices of all consumption categories. If prices are sticky, monetary policy then must trade off relative price distortions within different categories of consumption. Optimally, more weight should be placed on stabilizing goods and services prices that are less flexible. Calibrating a simple sticky-price model to U.S. data, we find that slight deflation is optimal, even absent transactions frictions leading to a demand for money. Optimality of deflation derives from the fact that relative prices have been trending up for services, whose nominal prices seem to be less flexible.

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1. Introduction

Analyses of the optimal rate of inflation typically use one-sector models. The two nonneutralities of money that are most frequently considered in these analyses involve transactions frictions which can be alleviated by holding money, and price stickiness. Transactions frictions lead almost inevitably to optimal deflation – the Friedman rule – whereas price stickiness typically leads to optimality of approximately stable prices.[1] Among policymakers and to some extent among researchers, a consensus has been reached that the transactions costs associated with approximately stable prices are low. Together with the benefits of stable prices with respect to price stickiness, this has led to a corresponding consensus that over the long run, approximately stable prices are the right objective for monetary policy to target.

One-sector models are the obvious starting point in macroeconomics, and for some issues they may be sufficient. Whether they are sufficient for determining the optimal rate of inflation is the question considered here. The inflation rate is an aggregate of the rates of price change for many goods and services. In turn, there is heterogeneity in those individual rates of price change. This heterogeneity means that stabilization of the price level cannot achieve stabilization of all individual prices. But stabilization of *individual prices* is what is optimal in sticky price models. To the extent that all individual prices cannot be stabilized because of heterogeneity that is exogenous with respect to monetary policy, it is not obvious that stabilization of the price level will be optimal. This issue has been studied in the context of cyclical fluctuations, by Huang and Liu [2005], Benigno [2004], Erceg and Levin [2006], and Aoki [2001].[2] Our concern here is the average inflation rate. Heterogeneity of price changes across sectors – or categories of consumption – is a trend phenomenon as well as a cyclical one. As such, the monetary authority cannot achieve zero average rates of price change for all consumption goods. If prices are sticky then it is infeasible to eliminate distortions associated with price dispersion, even in the absence of shocks.[3]

In a simple model with two consumption goods, I study the determinants of

[1] When we refer to price stickiness, we refer to some inflexibility in the level of nominal prices, in contrast to the assumption of indexation. With indexation the steady state inflation rate is irrelevant.

[2] Carlstrom, Fuerst and Ghironi [2006] study the conditions under which monetary policy rules generate determinacy in a two-sector model.

[3] Shirota [2007] performs a similar analysis in a model with an input-output structure, and Wolman [forthcoming] provides a survey on monetary policy with relative price variability.

the optimal rate of inflation when there is an exogenous trend in the relative price of the two goods. That trend results from trends in relative productivities for producing the two goods. I find that the principle determinant of the optimal inflation rate is interaction between the relative price trend and differential price stickiness across sectors. It is optimal to require less price adjustment of goods whose prices adjust less frequently. The expenditure-share weighted inflation rate is then disproportionately influenced by the relative price changes of those goods with more flexible prices.

Using personal consumption price and quantity data for the United States, I calibrate the technology processes and compute the optimal inflation rate. Optimality implies *deflation* at around four-tenths of a percent per year. Deflation is optimal because I assume that prices adjust less frequently in the sector with an increasing relative price. This assumption is consistent with the description of U.S. data in Bils and Klenow [2004]: prices of services adjust less frequently than prices of goods, and the relative price of services is rising over time. Imposing less price adjustment on services means that goods prices must fall at a greater rate than services prices rise, implying deflation overall.

The theme of this paper is related to Bosworth's [1980] description of a dilemma for monetary policy in the 1970s. According to Bosworth, positive relative price shocks for flexible price goods (commodities) were accommodated, letting the flexible prices rise in nominal terms while allowing the sticky-price goods to keep prices fixed. This was appropriate in that it minimized the distortions associated with price stickiness. Unfortunately it resulted in a higher inflation rate, and the lack of monetary policy credibility meant that expected inflation rose along with the actual inflation rate.[4] The 1970s involved a level-change rather than a trend in relative prices, but the policy response resulted in an inappropriate persistent increase in inflation. That episode suggests an important caveat to the technical analysis in this paper: while there may be sound theoretical arguments for deviating from zero inflation, in practice those deviations may have unintended consequences. Thus, policymakers should require a high threshold of evidence before purposefully deviating from near-zero inflation.

In the next section, I briefly describe U.S. data on price changes by consumption sector, as motivation for the modeling that follows. Section 3 lays out a flexible price model in order to provide basic intuition for how I generate the trending relative price and what some of its implications are. Section 4 describes

[4] The discussion of credibility and expected inflation is not in Bosworth [1980], but seems like a natural updating of Bosworth's interpretation given the work of the last twenty-five years.

the sticky price model, which is then used in section 5 to compute the optimal rate of inflation. Section 6 concludes and speculates on related topics and extensions to the framework used here.

2. Inflation and Sectoral Price Changes

The empirical inflation rate I focus on is that associated with the price index for personal consumption expenditure in the United States. Henceforth I will refer to this index as the PCE price index, and to its rate of change as PCE inflation. PCE inflation data are constructed by the Commerce Department's Bureau of Economic Analysis from underlying price and quantity data for a large number of categories of goods and services. In turn, the price data for those underlying categories are constructed from more direct observation of prices on an even larger number of specific items (i.e., goods and services). The latter construction is performed mainly by the Bureau of Labor Statistics in the Department of Labor. For the most part, the same item prices that form the basis for PCE inflation also form the basis for the more widely known CPI inflation, which is produced by the Bureau of Labor Statistics. I focus here on PCE inflation because the methodology used to produce the PCE inflation numbers corresponds more closely to notions of price indices suggested by economic theory.

Figure 1 plots the price indices for the three first-level components of personal consumption expenditure (durable goods, nondurable goods and services), together with the overall PCE price index. Each component differs somewhat from overall inflation. Services prices have risen much more than the overall index, averaging 4.2 percent compared to 3.42 percent for overall inflation. Durables price changes have generally been below PCE inflation, averaging 1.4 percent. The main distinguishing feature of nondurables price changes – which have averaged 3.2 percent – is that they have been more volatile than PCE inflation. Figure 1 shows that the differences in rates of price change across sectors have cumulated significantly over time: the price index for services has risen by a factor of more than 12 since 1947, whereas the price index for durables has risen by less than a factor of three.

Figure 2 plots expenditure shares for durable goods, nondurable goods, and services from 1947 to the present. Whereas the expenditure share for durable goods has fluctuated narrowly, between 10 and 18 percent, the shares of nondurables and services have respectively risen and fallen dramatically. In the first quarter of 1947 services accounted for only 31%, and nondurable goods accounted for 56% of personal consumption expenditure. In the first quarter of 2008 services accounted for 60% and nondurable goods for only 29% of personal consumption expenditure. The expenditure shift is striking, but it is not something I address in this paper.[5] Instead, I focus on the fact that in the last ten to fifteen years the expenditure shares appear to have stabilized, at around 11 percent for durable goods, 29 percent for nondurable goods, and 60 percent for services. It is this "stationary" period that generates the facts used to calibrate the model below. Note that although expenditure shares have stabilized, the trend in relative prices has not disappeared.

A maintained assumption in this paper is that monetary policy has perfect ability to control the inflation rate and no ability to affect relative prices across sectors. These assumptions are extreme and probably incorrect, but I believe they are useful for thinking about the optimal average inflation rate in an environment where there are trends in relative prices. Trends in relative prices are apparent in Figure 1, and Figure 3 illustrates them from a different perspective. Figure 3 displays "zero-inflation" price indexes for each category of consumption relative to the overall price index. That is, the lines in Figure 3 represent the component price indexes that would have yielded a zero overall inflation rate each quarter, assuming that relative prices and expenditure shares followed their historical paths. This figure shows that a zero inflation policy since 1947 would have implied trend decreases in both durable and nondurable goods prices, and a trend increase in services prices. In one-sector sticky price models, zero inflation eliminates the relative price distortion on average. With multiple sectors and relative price trends, zero inflation can be consistent with large relative price distortions.

3. A Competitive, Flexible Price Model

In this section I present a model with two sectors, each producing a distinct final consumption good (or view them as a good and a service). Production of the two goods is potentially subject to different rates of technological progress. One response to the data in Figure 2 might be that one should not be looking for an equilibrium with constant expenditure shares. Over the last ten years, however, it appears that expenditure shares have stabilized even as relative prices have trended more sharply. Just as importantly, analyzing steady states is a

[5] Ding and Wolman [2005] investigate whether shifting expenditure shares can help explain the time series properties of inflation. Greenwood and Uysal [2004] propose a model in which expenditure shares shift with the introduction of new goods.

much more straightforward exercise than analyzing equilibria in which a sector disappears asymptotically.

For optimal policy purposes I want to study a sticky-price version of the model. However, the steady state relationships are messy enough that it will be useful to work through the flexible price model first. In the flexible price version the monetary policy problem that motivates us is absent. The model consists of an infinitely lived representative household and two representative competitive firms. Households are endowed with labor and have preferences over the goods produced by the two firms. Each firm produces goods using a technology that is linear in labor. The firms hire labor from households in a competitive economywide labor market.

3.1. Households

The utility function of the representative household over a consumption index c_t and leisure l_t is

$$\{\sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j})\}, \tag{3.1}$$

with

$$u(c_t, l_t) = \log(c_t) + \chi l_t. \tag{3.2}$$

Labor supplied by households is $n_t = 1 - l_t$. The consumption-leisure trade-off yields

$$w_t = \chi c_t, \tag{3.3}$$

where w_t is the real wage. The consumption index is a Cobb-Douglas aggregate of $c_{g,t}$ and $c_{s,t}$, which are consumption of the two goods ("good" and "service"):

$$c_t = c_{g,t}^{\nu} c_{s,t}^{(1-\nu)}, \ \nu \in (0,1). \tag{3.4}$$

Optimal choices of the two goods, which have prices $P_{g,t}$ and $P_{s,t}$, solve the following problem:

$$\max \left[c_{g,t}^{\nu} c_{s,t}^{(1-\nu)} \right]$$
$$s.t. \ P_{g,t} c_{g,t} + P_{s,t} c_{s,t} = E,$$

where E is nominal expenditure, taken as given for the moment. The implied demands are

$$c_{g,t} = \nu \left(P_{g,t}/P_t\right)^{-1} c_t \tag{3.5}$$

and

$$c_{s,t} = (1-\nu)\left(P_{s,t}/P_t\right)^{-1} c_t. \tag{3.6}$$

The price level for the consumption composite is

$$P_t = \left(\frac{P_{g,t}}{\nu}\right)^{\nu} \left(\frac{P_{s,t}}{1-\nu}\right)^{1-\nu}. \tag{3.7}$$

Money demand is given exogenously by

$$M_t = P_{g,t}c_{g,t} + P_{s,t}c_{s,t},$$

which implies

$$M_t = P_t c_t.$$

Interaction of money demand with sectoral heterogeneity is an interesting topic, but I do not pursue it here.

3.2. Firms

Firms produce output according to a linear technology, and there are sector-specific productivities. So, for the two types of firms, the production functions are

$$c_{g,t} = z_{g,t}n_{g,t} \text{ and} \tag{3.8}$$

$$c_{s,t} = z_{s,t}n_{s,t}, \tag{3.9}$$

where $z_{g,t}$ and $z_{n,t}$ are the productivity levels, $n_{g,t}$ and $n_{s,t}$ are labor supplied to the two sectors, and $n_t = n_{g,t} + n_{s,t}$. With competitive labor and product markets, the common wage equals the marginal product of labor in each sector,

$$w_t = z_{g,t}P_{g,t}/P_t \tag{3.10}$$

$$w_t = z_{s,t}P_{s,t}/P_t, \tag{3.11}$$

and thus relative technologies pin down the relative price of one good in terms of the other:

$$\frac{P_{g,t}}{P_{s,t}} = \frac{z_{s,t}}{z_{g,t}}. \tag{3.12}$$

3.3. A Steady State (possibly with inflation)

In a steady state there is constant inflation, constant real growth of the aggregate c, and a constant trend in the price ratio of the two goods. In order for there to be a steady state equilibrium, growth rates of the exogenous money supply and of the exogenous technology parameters must be constant.

I use the following notation for growth rates, normalizing all exogenous variables to equal 1 when $t = 0$:

$$\begin{aligned} M_t &= (1+\mu)^t \\ z_{g,t} &= \left(1+\zeta_g\right)^t \\ z_{s,t} &= (1+\zeta_s)^t . \end{aligned} \tag{3.13}$$

Expenditure shares of the two goods are constant:

$$\begin{aligned} \frac{P_g c_g}{Pc} &= \nu \\ \frac{P_s c_s}{Pc} &= 1-\nu. \end{aligned}$$

One can easily determine the values of all other variables in steady state. Combining the labor supply equation (3.3) with (3.10), then using (3.12) and (3.7), we have

$$c_t = \chi^{-1}\left(\nu^\nu (1-\nu)^{1-\nu}\right)\left(1+\zeta_g\right)^{\nu t}(1+\zeta_s)^{(1-\nu)t} .$$

The real wage is then

$$w_t = \left(\nu^\nu (1-\nu)^{1-\nu}\right)\left(1+\zeta_g\right)^{\nu t}(1+\zeta_s)^{(1-\nu)t} . \tag{3.14}$$

From (3.10) and (3.11), prices of the two goods relative to the overall price index are

$$P_{g,t}/P_t = \left(\nu^\nu (1-\nu)^{1-\nu}\right)\left(\frac{1+\zeta_s}{1+\zeta_g}\right)^{(1-\nu)t} .$$

and

$$P_{s,t}/P_t = \left(\nu^\nu (1-\nu)^{1-\nu}\right)\left(\frac{1+\zeta_g}{1+\zeta_s}\right)^{\nu t} .$$

Finally, using (3.5) and (3.6) output of the two goods is given by

$$c_{g,t} = (\nu/\chi)\left(1+\zeta_g\right)^t, \tag{3.15}$$

and

$$c_{s,t} = \left(\frac{1-\nu}{\chi}\right)(1+\zeta_s)^t. \tag{3.16}$$

As for nominal variables, the price level P_t grows at the rate of money growth divided by the rate of consumption growth:

$$1+\pi = \frac{1+\mu}{\left(1+\zeta_g\right)^{\nu}(1+\zeta_s)^{1-\nu}}, \tag{3.17}$$

the sector g price $P_{g,t}$ grows at rate $(1+\mu)/\left(1+\zeta_g\right)$, and the sector s price grows at rate $(1+\mu)/(1+\zeta_s)$. The growth rate of P_t is the model's true inflation rate– that is, the rate of change of the price of one unit of c. United States PCE inflation is well approximated by an expenditure-share weighted average of the price changes for different categories of goods and services. We can easily compute the model's equivalent of PCE inflation in the same manner:

$$1+\pi_{pce} = (1+\mu)\left[\frac{\nu}{1+\zeta_g}+\frac{1-\nu}{1+\zeta_s}\right]. \tag{3.18}$$

Because it is the standard empirical measure, I will emphasize the PCE inflation rate in the numerical results below, though I will also report the true inflation rate. As long as the productivity differential across sectors is not huge, PCE inflation will be a good approximation to the true inflation rate.

4. The Model with Sticky Prices

In the flexible price model presented above, the strongest form of monetary neutrality holds. It does not "matter" that prices of different goods change at different rates in a steady state. Now suppose that each sector is made up of a large number of monopolistically competitive firms, and that there is some sort of staggered price setting in each sector. In this case, different nominal rates of price change across sectors mean that it is infeasible for the monetary authority to stabilize all nominal prices. This type of problem is familiar from the work of Erceg, Henderson and Levin [2000] (for sticky nominal wages and prices), Erceg and Levin [2006] Huang and Liu [2005], and Aoki [2001]. However, each of those papers is concerned with short run fluctuations in relative prices.[6] If prices are indeed sticky in nominal terms, so that there is a relative price distortion associated with non-zero steady state inflation, then the optimal steady state rate of inflation represents a nontrivial problem for the monetary authority. In this section I describe a sticky-price version of the model presented above. I reinterpret $c_{g,t}$ and $c_{s,t}$ as aggregates of a continuum of monopolistically produced goods whose prices are less than perfectly flexible.

4.1. Households

Up until the definition of the good and service, the description of household behavior is unchanged. Now however, the good and service are composites made up of Dixit-Stiglitz aggregates of a continuum of differentiated products, with elasticity of substitution ε:

$$c_{k,t} = \left[\int_0^1 c_{k,t}(z)^{(\varepsilon-1)/\varepsilon} dz\right]^{\varepsilon/(\varepsilon-1)}, \quad \text{for } k = g, s,$$

which leads to the demands

$$c_{k,t}(z) = \left(\frac{P_{k,t}(z)}{P_{k,t}}\right)^{-\varepsilon} c_{k,t}, \quad \text{for } k = g, s. \tag{4.1}$$

Combining the two levels of demands (4.1) and (3.5) yields

$$c_{g,t}(z) = \nu \left(\frac{P_{g,t}(z)}{P_{g,t}}\right)^{-\varepsilon} \left(\frac{P_{g,t}}{P_t}\right)^{-1} c_t \tag{4.2}$$

and

$$c_{s,t}(z) = (1-\nu) \left(\frac{P_{s,t}(z)}{P_{s,t}}\right)^{-\varepsilon} \left(\frac{P_{s,t}}{P_t}\right)^{-1} c_t. \tag{4.3}$$

The price level for the consumption aggregate is

$$P_t = \left(\frac{P_{g,t}}{\nu}\right)^{\nu} \left(\frac{P_{s,t}}{1-\nu}\right)^{1-\nu}, \tag{4.4}$$

[6] Loyo [2002] points out that if there were multiple units of account, nominal item prices could remain constant while the relative price of the units of account did the adjusting, along the lines of Friedman's case for flexible exchange rates. In the same way, multiple units of account could eliminate the problem with trending relative prices addressed here.

and the price indexes for the composite good and service are

$$P_{k,t} = \left[\int_0^1 \left[P_{k,t}\left(z\right) \right]^{1-\varepsilon} dz \right]^{1/(1-\varepsilon)}, \text{ for } k = g, s.$$

Substituting into the price index for the consumption aggregate, we have

$$P_t = \left(\frac{\left[\int_0^1 \left[P_{g,t}\left(z\right) \right]^{1-\varepsilon} dz \right]^{1/(1-\varepsilon)}}{\nu} \right)^{\nu} \left(\frac{\left[\int_0^1 \left[P_{s,t}\left(z\right) \right]^{1-\varepsilon} dz \right]^{1/(1-\varepsilon)}}{1-\nu} \right)^{1-\nu}.$$

Now I incorporate price stickiness, in a form which is a generalization of Calvo and Taylor style time-dependent pricing.[7] A firm in sector k will adjust its price with (exogenous) probability $\alpha_{k,j}$, where j indexes the number of periods since the last price adjustment. The set of price adjustment probabilities $\alpha_{k,j}$, $j = 1, ..., J_k - 1$ are collected in the vector $\underset{\sim}{\alpha}_k$, for $k = g, s$, where J_k is the maximum number of periods that a firm in sector k will remain with the same price. From the vector $\underset{\sim}{\alpha}_k$ one can derive the fractions of firms in period t charging prices set in periods $t - j$, which will be denoted by $\omega_{k,j}$. To do this, note that

$$\begin{aligned} \omega_{k,j} &= \left(1 - \alpha_{k,j}\right) \omega_{k,j-1}, \text{ for } j = 1, 2, ..., J_k - 1, \\ &\quad \text{and} \\ \omega_{k,0} &= 1 - \sum_{j=1}^{J_k - 1} \omega_{k,j}. \end{aligned} \tag{4.5}$$

This system of linear equations can be solved for $\omega_{k,j}$ as a function of $\alpha_{k,j}$. With this specification of price stickiness, and the further assumptions that (1) there is a stationary distribution of firms in terms of the age of their price, and (2) all firms of the same vintage charge the same price, rewrite the price index as follows:

$$P_t = \left(\frac{\left[\sum_{j=0}^{J_g - 1} \omega_{g,j} P_{g,j,t}^{1-\varepsilon} \right]^{1/(1-\varepsilon)}}{\nu} \right)^{\nu} \left(\frac{\left[\sum_{j=0}^{J_s - 1} \omega_{s,j} P_{s,j,t}^{1-\varepsilon} \right]^{1/(1-\varepsilon)}}{1-\nu} \right)^{1-\nu}$$

where $P_{k,j,t}$ denotes the price charged in period t by a firm in sector k that last adjusted its price in period $t - j$. Note that $P_{k,j,t} = P_{k,0,t-j}$.

[7] See Wolman [1999] for more details.

Assume as before that households also hold money equal to nominal expenditure,

$$M_t = \int_0^1 P_{g,t}(z) c_{g,t}(z)\, dz + \int_0^1 P_{s,t}(z) c_{s,t}(z)\, dz. \tag{4.6}$$

With constant-elasticity demands for each good, the money-demand specification in (4.6) implies

$$M_t = P_t c_t. \tag{4.7}$$

Let λ_t be the Lagrange multiplier which represents the shadow value of wealth,

$$\lambda_t = \frac{1}{c_t}. \tag{4.8}$$

Households equate the marginal rate of substitution between leisure and consumption to the competitive real wage rate:

$$w_t = \chi c_t. \tag{4.9}$$

4.2. Firms

Firms produce output according to a linear technology, and there are sector-specific productivities. So, for each type of firm, the production function is

$$c_{k,j,t} = z_{k,t} n_{k,j,t}, \text{ for } k = g, s. \tag{4.10}$$

This implies that real marginal cost is unrelated to the scale of the firm and is simply

$$\psi_{k,t} = w_t / z_{k,t}, \text{ for } k = g, s,$$

and that nominal marginal cost is

$$\Psi_{k,t} = P_t w_t / z_{k,t}, \text{ for } k = g, s.$$

Firms that adjust their prices in period t set prices so as to maximize the expected present discounted value of their profits, using the household's marginal utility as a discount factor. That is, they choose $P_{k,0,t}$ to maximize their market value,

$$\sum_{j=0}^{J_k - 1} \beta^j \omega_{j,k} \lambda_{t+j} \left(\frac{P_{k,0,t} - \Psi_{k,t+j}}{P_{t+j}} \right) c_{k,j,t+j}.$$

As monopolistic competitors, firms face demands given by (4.2) and (4.3). Thus the price-setting problem for a firm in sector j is to maximize

$$\sum_{j=0}^{J_k-1} \beta^j \omega_{j,k} \lambda_{t+j} \left(\frac{P_{k,0,t} - \Psi_{k,t+j}}{P_{t+j}} \right) \nu^k \left(\frac{P_{k,0,t}}{P_{k,t+j}} \right)^{-\varepsilon} \left(\frac{P_{k,t+j}}{P_{t+j}} \right)^{-1} c_{t+j}$$

(note the abuse of notation now: ν^k is either ν or $(1-\nu)$). The first-order condition for this problem can be manipulated to isolate the relative price chosen by the adjusting firm:

$$\frac{P_{k,0,t}}{P_t} = \frac{\varepsilon \sum_{j=0}^{J_k-1} \beta^j \omega_{j,k} \lambda_{t+j} \left(\frac{\Psi_{k,t+j}}{P_{t+j}} \right) \nu^k \left(\frac{P_{k,t}}{P_{k,t+j}} \right)^{-\varepsilon} \left(\frac{P_{k,t+j}}{P_{t+j}} \right)^{-1} c_{t+j}}{(\varepsilon - 1) \sum_{j=0}^{J_k-1} \beta^j \omega_{j,k} \lambda_{t+j} \frac{P_t}{P_{t+j}} \nu^k \left(\frac{P_{k,t}}{P_{k,t+j}} \right)^{-\varepsilon} \left(\frac{P_{k,t+j}}{P_{t+j}} \right)^{-1} c_{t+j}}.$$

Now replace nominal marginal cost with real marginal cost multiplied by the price level and use the fact that $\lambda_{t+j} c_{t+j} = 1$:

$$\frac{P_{k,0,t}}{P_t} = \frac{\varepsilon \sum_{j=0}^{J_k-1} \beta^j \omega_{j,k} \psi_{k,t+j} \nu^k \left(\frac{P_{k,t}}{P_{k,t+j}} \right)^{-\varepsilon} \left(\frac{P_{k,t+j}}{P_{t+j}} \right)^{-1}}{(\varepsilon - 1) \sum_{j=0}^{J_k-1} \beta^j \omega_{j,k} \frac{P_t}{P_{t+j}} \nu^k \left(\frac{P_{k,t}}{P_{k,t+j}} \right)^{-\varepsilon} \left(\frac{P_{k,t+j}}{P_{t+j}} \right)^{-1}}.$$

In order to make progress toward a steady state, use the facts that $\psi_{k,t} = w_t / z_{k,t}$ and $w_t = \chi c_t$. Together these imply

$$\psi_{k,t} = \chi c_t / z_{k,t}$$

and thus

$$\frac{P_{k,0,t}}{P_t} = \frac{\varepsilon \sum_{j=0}^{J_k-1} \beta^j \omega_{j,k} \chi \left(c_{t+j} / z_{k,t+j} \right) \left(\frac{P_{k,t}}{P_{k,t+j}} \right)^{-\varepsilon} \left(\frac{P_{k,t+j}}{P_{t+j}} \right)^{-1}}{(\varepsilon - 1) \sum_{j=0}^{J_k-1} \beta^j \omega_{j,k} \frac{P_t}{P_{t+j}} \left(\frac{P_{k,t}}{P_{k,t+j}} \right)^{-\varepsilon} \left(\frac{P_{k,t+j}}{P_{t+j}} \right)^{-1}}, \text{ for } k = g, s. \quad (4.11)$$

4.3. Inflationary Steady State

The next step is to derive a steady state, in order to compute welfare for a range of money-growth rates (and hence inflation rates). I use the same notation for the exogenous processes as described in (3.13). The principal equations used to derive a steady state are the two optimal pricing equations (4.11), the two price indices,

$$P_{k,t} = \left\{ \sum_{j=0}^{J_k-1} \omega_{k,j} P_{k,j,t}^{1-\varepsilon} \right\}^{1/(1-\varepsilon)}, \quad \text{for } k = g, s; \tag{4.12}$$

and the overall price index (4.4). In this section I use these equations to derive the model's steady state. The first two subsections below manipulate the optimal pricing equations and the price indices, respectively. The third subsection completes the derivations.

4.3.1. Optimal Pricing

Define the rates of price change in steady state as follows:

$$\begin{aligned} 1+\pi &\equiv P_t/P_{t-1}, \\ 1+\pi_g &\equiv P_{g,0,t}/P_{g,0,t-1}, \text{ and} \\ 1+\pi_s &\equiv P_{s,0,t}/P_{s,0,t-1}. \end{aligned} \tag{4.13}$$

Note that together with the sectoral price index equations, these definitions imply

$$\begin{aligned} P_{g,t}/P_{g,t-1} &= 1+\pi_g, \text{ and} \\ P_{s,t}/P_{s,t-1} &= 1+\pi_s. \end{aligned} \tag{4.14}$$

That is, if a firm adjusting its price in $t+1$ sets a price that is a multiple $1+\pi_g$ of the price set by a firm that adjusted in t, then the sectoral price index also increases at rate $1+\pi_g$.

Then rewrite the optimal pricing equations, using these definitions, using the assumptions about technology shifters in (3.13), and noting that the growth rate of consumption is the product of the discount factor and the real interest rate:

$$c_t/c_{t-1} = \beta\,(1+r)\,.$$

The optimal pricing equations become

$$\frac{P_{k,0,t}}{P_t} = \frac{\varepsilon\chi c_0}{(\varepsilon-1)} \cdot \left(\frac{\beta\,(1+r)}{1+\zeta_k}\right)^t \frac{\sum_{j=0}^{J_k-1} \beta^j \omega_{j,k} \left(\frac{\beta(1+r)}{1+\zeta_k}\right)^j (1+\pi_k)^{j\varepsilon} \left(\frac{1+\pi}{1+\pi_k}\right)^j}{\sum_{j=0}^{J_k-1} \beta^j \omega_{j,k} \,(1+\pi_k)^{j(\varepsilon-1)}}, \quad \text{for } k = g, s; \tag{4.15}$$

4.3.2. From the price indices

Using the shorthand for rates of price change in (4.14), the industry price indices can be rewritten as

$$\frac{P_{k,t}}{P_{k,0,t}} = \left\{ \sum_{j=0}^{J_k - 1} \omega_{k,j} \left(1 + \pi_k\right)^{j(\varepsilon - 1)} \right\}^{1/(1-\varepsilon)}, \quad \text{for } k = g, s. \tag{4.16}$$

Substituting these expressions into the overall price index yields a relationship between relative prices in the two sectors:

$$1 = \left(\left(\frac{P_{g,0,t}}{P_t}\right) \frac{\left[\sum_{j=0}^{J_g - 1} \omega_{g,j} \left(1 + \pi_g\right)^{-j(1-\varepsilon)}\right]^{1/(1-\varepsilon)}}{\nu} \right)^{\nu} \left(\left(\frac{P_{s,0,t}}{P_t}\right) \frac{\left[\sum_{j=0}^{J_s - 1} \omega_{s,j} \left(1 + \pi_s\right)^{-j(1-\varepsilon)}\right]^{1/(1-\varepsilon)}}{1 - \nu} \right)^{1-\nu}. \tag{4.17}$$

Time does not appear on the LHS of this expression, thus, it must be that time does not appear on the RHS either. This implies that the trends in $\left(\frac{P_{g,0,t}}{P_t}\right)^{\nu}$ and $\left(\frac{P_{s,0,t}}{P_t}\right)^{1-\nu}$ are offsetting, and from (4.15) those trends satisfy

$$\left(\frac{P_{g,0,t}}{P_t}\right)^{\nu} \left(\frac{P_{s,0,t}}{P_t}\right)^{1-\nu} \propto \left(\beta \left(1 + r\right) \left(1 + \zeta_g\right)^{-\nu} \left(1 + \zeta_s\right)^{\nu - 1}\right)^{t}. \tag{4.18}$$

Imposing lack of a time trend on $\left(\frac{P_{g,0,t}}{P_t}\right)^{\nu} \left(\frac{P_{s,0,t}}{P_t}\right)^{1-\nu}$, the real interest rate is

$$\left(1 + r\right) = \beta^{-1} \left(1 + \zeta_g\right)^{\nu} \left(1 + \zeta_s\right)^{1-\nu}. \tag{4.19}$$

4.3.3. Completing the Steady State Derivation

Recalling that the money growth rate is $1 + \mu$, from the money demand equation we have

$$\left(1 + r\right) = \beta^{-1} \left(\frac{1 + \mu}{1 + \pi}\right). \tag{4.20}$$

Thus from (4.19) and (4.20) we have the inflation rate:

$$1+\pi = \frac{(1+\mu)}{\left(1+\zeta_g\right)^{\nu}\left(1+\zeta_s\right)^{1-\nu}}. \tag{4.21}$$

It remains to compute the sectoral rates of price change. From (4.15) we have

$$\frac{P_{k,0,t}}{P_{k,0,t-1}} = (1+\pi)\left(\frac{\beta(1+r)}{1+\zeta_k}\right)$$

which implies

$$\frac{P_{g,0,t}}{P_{g,0,t-1}} = 1+\pi_g = \left(\frac{1+\mu}{1+\zeta_g}\right) \tag{4.22}$$

$$\frac{P_{s,0,t}}{P_{s,0,t-1}} = 1+\pi_s = \left(\frac{1+\mu}{1+\zeta_s}\right). \tag{4.23}$$

Relative price levels also need to be computed. The rates of change of relative prices are given by the last three numbered equations. The relative price levels in period zero come from (4.15), as functions of c_0 :

$$\frac{P_{k,0,0}}{P_0} = \frac{\varepsilon\chi c_0}{(\varepsilon-1)}\cdot\frac{\sum_{j=0}^{J_k-1}\beta^j\omega_{j,k}\left(\frac{1+\mu}{1+\zeta_k}\right)^{j\varepsilon}}{\sum_{j=0}^{J_k-1}\beta^j\omega_{j,k}\left(\frac{1+\mu}{1+\zeta_k}\right)^{j(\varepsilon-1)}}, \quad \text{for } k=g,s. \tag{4.24}$$

There are now three equations ((4.24) for $k=g,s$ and (4.17)) in the three variables c_0, $\frac{P_{g,0,0}}{P_0}$ and $\frac{P_{s,0,0}}{P_0}$. To solve for c_0, substitute (4.24) into (4.17) (also eliminate $1+\pi$ using our solution):

$$c_0^{-1} = \chi\left(\frac{\varepsilon}{(\varepsilon-1)}\cdot\frac{\sum\limits_{j=0}^{J_g-1}\beta^j\omega_{j,g}\left(\frac{1+\mu}{1+\zeta_g}\right)^{j\varepsilon}}{\sum\limits_{j=0}^{J_g-1}\beta^j\omega_{j,g}\left(\frac{1+\mu}{1+\zeta_g}\right)^{j(\varepsilon-1)}}\frac{\left[\sum\limits_{j=0}^{J_g-1}\omega_{g,j}\left(\frac{1+\mu}{1+\zeta_g}\right)^{\frac{-j}{1-\varepsilon}}\right]^{\frac{1}{1-\varepsilon}}}{\nu}\right)^{\nu} \tag{4.25}$$
$$\left(\frac{\varepsilon}{(\varepsilon-1)}\cdot\frac{\sum\limits_{j=0}^{J_s-1}\beta^j\omega_{j,s}\left(\frac{1+\mu}{1+\zeta_s}\right)^{j\varepsilon}}{\sum\limits_{j=0}^{J_s-1}\beta^j\omega_{j,s}\left(\frac{1+\mu}{1+\zeta_s}\right)^{j(\varepsilon-1)}}\frac{\left[\sum\limits_{j=0}^{J_s-1}\omega_{s,j}\left(\frac{1+\mu}{1+\zeta_s}\right)^{\frac{-j}{1-\varepsilon}}\right]^{\frac{1}{1-\varepsilon}}}{1-\nu}\right)^{1-\nu}.$$

The last step is to compute leisure:

$$l_t = 1 - n_t.$$

Labor input (n_t) is the weighted sum of labor input for every type of firm, where the weights are the fractions of firms:

$$n_t = \sum_{j=0}^{J_g-1} \omega_{g,j} \frac{c_{g,j,t}}{z_{g,t}} + \sum_{j=0}^{J_s-1} \omega_{s,j} \frac{c_{s,j,t}}{z_{s,t}}.$$

Using the demand functions (4.2) and (4.3),

$$\begin{aligned} n_0 \;=\; c_0 \Bigg[\nu \left(\frac{P_{g,0}}{P_0}\right)^{-1} \left(\frac{P_{g,0,0}}{P_{g,0}}\right)^{-\varepsilon} \sum_{j=0}^{J_g-1} \omega_{g,j} \left(1+\pi_g\right)^{j\varepsilon} + \\ (1-\nu) \left(\frac{P_{s,0}}{P_0}\right)^{-1} \left(\frac{P_{s,0,0}}{P_{s,0}}\right)^{-\varepsilon} \sum_{j=0}^{J_s-1} \omega_{s,j} \left(1+\pi_s\right)^{j\varepsilon} \Bigg]. \end{aligned} \tag{4.26}$$

To compute n_0 using this last equation, use the price ratios $\frac{P_{k,0,0}}{P_{k,0}}$ from (4.25) and (4.24) and get the other price ratios $\frac{P_{k,0}}{P_0}$ from

$$\frac{P_{k,0}}{P_0} = \frac{P_{k,0,0}}{P_0} \div \frac{P_{k,0,0}}{P_{k,0}}, \tag{4.27}$$

where the first factor comes from (4.24) and the second factor comes from (4.16):

$$\frac{P_{k,0,0}}{P_{k,0}} = \left\{ \sum_{j=0}^{J_k-1} \omega_{k,j} \left(\frac{1+\mu}{1+\zeta_k}\right)^{j(\varepsilon-1)} \right\}^{1/(\varepsilon-1)}, \quad \text{for } k = g, s. \tag{4.28}$$

The steady state derivation is now complete in that for given parameter values steady state welfare follows from the solutions for c_t and n_t.

5. The Optimal Rate of Inflation

The steady state money growth rate – and hence the steady state rate of inflation – matter in this model for two reasons. First, with sticky prices, different average inflation rates correspond to different levels of the monopoly markup in both sectors. Second, and more importantly from the perspective of our motivation, sticky prices mean that different average inflation rates correspond to different degrees of relative price distortion within each sector. Here I describe the two sets of distortions and show how they summarize the channels through which monetary policy can affect steady state welfare.

5.1. Sectoral Markups

The sectoral markup ($\mathcal{M}$) of price over marginal cost in sector k is given by

$$\begin{aligned} \mathcal{M}_k &= \frac{P_{k,0}}{P_0 w_0} = \frac{P_{k,0}}{\chi P_0 c_0} \\ &= (\chi c_0)^{-1} \left(\frac{P_{k,0}}{P_{k,0,0}} \frac{P_{k,0,0}}{P_0} \right). \end{aligned} \tag{5.1}$$

In a flexible price version of the model, the sectoral markup would simply be $\varepsilon / (\varepsilon - 1)$. With sticky prices, however, from (4.25), (4.28) and (4.24), it is clear that the sectoral markups depend on monetary policy (through the money growth rate μ):

$$\mathcal{M}_k = \frac{\varepsilon}{\varepsilon - 1} \cdot \left(\left\{ \sum_{j=0}^{J_k - 1} \omega_{k,j} \left(\frac{1+\mu}{1+\zeta_k} \right)^{j(\varepsilon-1)} \right\}^{1/(1-\varepsilon)} \frac{\sum_{j=0}^{J_k-1} \beta^j \omega_{j,k} \left(\frac{1+\mu}{1+\zeta_k} \right)^{j\varepsilon}}{\sum_{j=0}^{J_k-1} \beta^j \omega_{j,k} \left(\frac{1+\mu}{1+\zeta_k} \right)^{j(\varepsilon-1)}} \right),$$

$$\text{for } k = g, s. \tag{5.2}$$

With sticky prices, an *individual firm* would like to achieve the markup $\varepsilon / (\varepsilon - 1)$ in every period (note that $\mathcal{M}_k$ is the *aggregate* sectoral markup). However, if there is a trend in the price of the firm's output, then infrequent price adjustment means that the optimal markup cannot be achieved in every period. Firms optimally set a high markup (i.e. greater than $\varepsilon / (\varepsilon - 1)$) in the period they adjust, and then watch it depreciate as long as their price is fixed. For an individual firm, the markup when adjusting is increasing in the sectoral rate of price change. However, a greater positive rate of price change also causes the markup to depreciate faster for nonadjusting firms.[8] For the Calvo model (where $\omega_{k,j}$ declines geometrically with j), it is straightforward to show that together the two effects generate a markup that is convex in the rate of sectoral price change: $\mathcal{M}_k$ is minimized at a

[8] For additional discussion, see King and Wolman [1996].

small but positive value of sectoral price change (i.e. $\frac{1+\mu}{1+\zeta_g} > 1$). For the general time-dependent model, no such results are immediate. It is clear however, that if the price stickiness and demand elasticity parameters are common across sectors, but productivity growth is not, then the markup-minimizing money growth rates will differ across sectors. Thus, it will not be possible to choose steady state inflation in order to minimize the markup in both sectors.

5.2. Sectoral Relative Price Distortions

Relative price distortion refers to the fact that with sticky prices, if prices within a sector change over time at a constant rate, then at any point in time there will be more than one price charged by firms within the sector. Because firms charge different prices, they will also produce in different quantities. This is inefficient because of the symmetry in preferences and production within a sector: efficiency dictates that firms within a sector should produce in the same quantity. Define the relative price distortion within a sector as

$$\mathcal{R}_k = \frac{z_{k,t} n_{k,t}}{c_{k.t}}.$$

This has the interpretation of the ratio of (a), the amount of the sector k composite that *could be* produced with the current labor input in sector k to (b), the amount of the sector k composite that *is* produced by the current labor input in sector k.

The relative price distortion is influenced by monetary policy, because the rate of price change in the sector determines the dispersion of prices within the sector. Using the production functions and demand functions makes precise the relationship between monetary policy and the relative price distortion:

$$\begin{aligned} \mathcal{R}_k &= \frac{\sum_{j=0}^{J_k-1} \omega_{k,j} c_{k,j,t}}{c_{k.t}} \\ &= \left(\frac{P_{k,0,0}}{P_{k,0}}\right)^{-\varepsilon} \sum_{j=0}^{J_k-1} \omega_{k,j} \left(\frac{1+\mu}{1+\zeta_k}\right)^{j\varepsilon}, \end{aligned} \tag{5.3}$$

where $\frac{P_{k,0,0}}{P_{k,0}}$ is given by (4.28) for $k = g, s$. The relative price distortion is a convex function of $(1+\mu)$, minimized at $\mu = \zeta_k$. When $\mu = \zeta_k$, firms in sector k never need to change their prices ($\pi_k = 0$), so the stickiness of prices becomes unimportant and the relative price distortion is eliminated. This corresponds to zero inflation in a one-sector model. But in our two-sector model, if there are different rates of productivity growth in the two sectors ($\zeta_g \neq \zeta_s$) the relative price distortion cannot be eliminated in both sectors.

5.3. Optimal Steady State Inflation

If the rate of productivity growth differs across sectors, then both the markup and the relative price distortion require the monetary authority to trade off benefits to one sector against costs to the other sector when choosing the inflation rate. In fact, the level of steady state welfare can be expressed as a function of only the sectoral relative price distortions and markups.

Steady state welfare in period zero is

$$\begin{aligned} \mathrm{v}_0^{ss} &= \sum_{j=0}^{\infty} \beta^j \left\{ \ln(c_j) + \chi l_j \right\} \\ &= \sum_{j=0}^{\infty} \beta^j \left(j \ln\left((1+\zeta_s)(1+\gamma_s) \right) \right) + \sum_{j=0}^{\infty} \beta^j \left\{ \ln c_0 + \chi (1-n_0) \right\}. \end{aligned}$$

The first term is unaffected by monetary policy, so focus on the second term:

$$\tilde{\mathrm{v}}_0^{ss} = \sum_{j=0}^{\infty} \beta^j \left\{ \ln c_0 + \chi (1-n_0) \right\}.$$

From the expressions for c_0 (4.25) and the sectoral markups (5.2), c_0 can be written as a function of the sectoral markups:

$$c_0 = \chi^{-1} \left(\frac{\mathcal{M}_g}{\nu} \right)^{-\nu} \left(\frac{\mathcal{M}_s}{1-\nu} \right)^{\nu-1}.$$

Also, from expressions for n_t (4.26) and the sectoral relative price distortions (5.3), as well as the sectoral markups (5.1), n_0 can be written as a function of the markups and relative price distortions:[9]

$$\frac{P_{k,0}}{P_0} = \chi c_0 \mathcal{M}_k$$

$$n_0 = \chi^{-1} \left(\frac{\nu \mathcal{R}_g}{\mathcal{M}_g} + \frac{(1-\nu)\mathcal{R}_s}{\mathcal{M}_s} \right). \tag{5.4}$$

[9] Recall that $n_0 = n_t$.

When one combines the last two equations, it becomes clear that monetary policy affects steady state welfare only through its effects on the sectoral markups and relative price distortions (the following expression omits the constant term $1/\chi$):

$$\begin{aligned}\tilde{\mathrm{v}}_0^{ss}(\mu;.) &= \frac{1}{1-\beta}\{-\nu\ln\mathcal{M}_g(\mu;.)-(1-\nu)\ln\mathcal{M}_s(\mu;.)\\ &\quad-\left(\frac{\nu\mathcal{R}_g(\mu;.)}{\mathcal{M}_g(\mu;.)}+\frac{(1-\nu)\mathcal{R}_s(\mu;.)}{\mathcal{M}_s(\mu;.)}\right)\Bigg\}. \qquad (5.5)\end{aligned}$$

Recall that the productivity growth rates show up in the solutions for consumption, leisure and thus the distortions. Therefore, the optimal money growth rate is sensitive to the productivity growth rates.

6. Numerical Results

I will present two types of results. First I will look at a benchmark calibration in order to understand how the optimal inflation rate trades off the four distortions described above. Then I will vary the productivity processes and the degree of price stickiness to illustrate the sensitivity of the optimal inflation rate to those parameters.

6.1. Calibration

I interpret the model as describing quarterly data. Most of the model's parameters are fixed a priori: $\beta = 0.99$, $\chi = 4.5$, and $\varepsilon = 10$. The parameters representing price stickiness are chosen somewhat arbitrarily, but with some input from the results in Bils and Klenow, showing that goods prices change more frequently than services prices: I set $\underset{\sim}{\alpha}_g = 0$ and $\underset{\sim}{\alpha}_s = [0\,0]'$, meaning that goods prices are fixed for two quarters and services prices are fixed for three quarters.

The parameter ν is equal to the expenditure share of goods in total consumption. I set this parameter to 0.40, which was the 2008 expenditure share of goods in consumption.

Calibration of the productivity trends is based on the data in Section 2, along with corresponding data on real consumption expenditure. First I aggregate durable goods and nondurable goods price changes using their expenditure shares to get a rate of price change for goods. Over the period 1993 to 2008,

1. The price of goods relative to that of services 'grew' at a gross quarterly rate of 0.9956 (that is, it fell 0.44 percent per quarter).

2. Real personal consumption expenditure per capita grew at a gross quarterly rate of 1.0059 – about 0.6 percent per quarter.

I match up the relative price growth rate to the inverse of the relative productivity shifters' growth:

$$\frac{1+\zeta_s}{1+\zeta_g} = 0.9956. \tag{6.1}$$

I also match up real consumption growth in the data with the expenditure share weighted average of real growth in the two sectors, which gives the equation

$$1.0059 = \nu\left(1+\zeta_g\right) + (1-\nu)\left[0.9956\left(1+\zeta_g\right)\right].$$

Solving for ζ_g given $\upsilon = 0.40$ yields

$$1+\zeta_g = 1.00856.$$

The first equation (6.1) then determines ζ_s.

6.2. Benchmark Results

The calibration (and the U.S. data) involves a trend in the relative price of goods to services, and that trend is exogenous with respect to monetary policy. It is therefore impossible for the monetary authority to replicate the flexible price allocation by appropriate choice of the money growth rate. The relative price of c_g to c_s needs to change on average, so nominal prices within sector g or sector s (or both) must change on average. There will then be real effects of price stickiness.

Figure 4 plots welfare, the sectoral relative price distortions and markups, and the sectoral rates of price change as a function of the PCE inflation rate. In the background, the steady-state money growth rate μ is being varied to induce the changes in PCE inflation, according to (3.18). The optimal PCE inflation rate is approximately -0.4 percent, implying small deflation. Why deflation? The relative price of services will rise regardless of the inflation rate (see the top right panel of the figures). At zero inflation, the nominal price of goods will be falling at approximately the same rate as the nominal price of services is rising. With symmetric sectors - other than productivity growth– this would imply *approximately* equal relative price distortions in the two sectors.[10] Recall

[10] The relative price distortions in the two sectors would not be equal if the sectors were symmetric because price decreases have different effects on the markup than price increases of the same magnitude. We discuss this further in the next subsection.

however that c_g prices are more flexible than c_s prices. Relative flexibility in the g sector means that a given rate of price change translates into a smaller relative price distortion. It is optimal then for the g sector to bear more of the burden of nominal adjustment, and since the relative prices in the g sector are falling, this means that nominal prices in the g sector should fall more than nominal prices in the s sector should rise. Panel C illustrates the fundamental trade-off facing the monetary authority: that it cannot eliminate the relative price distortion in both sectors. The relative stickiness of s prices is reflected in panels C and D in the higher degree of curvature in the two distortions for the s sector compared to the g sector.

6.3. The Relative Price Trend and Differential Price Stickiness

Figures 5 and 6 illustrate how the optimal steady state inflation rate varies with the degree of price stickiness in sector s and the rate of productivity growth in sector s. Specifically, for Figure 5, I fix the degree of price stickiness in sector g as described above (two-quarter pricing) and vary the expected duration of prices in sector s from one quarter (flexible prices) to six quarters. Panels A and B of Figure 5 show that as price stickiness in sector s rises to even moderate levels, the optimal inflation rate becomes negative and nearly stabilizes the price level in the s sector, imposing the entire burden of relative price adjustment on nominal prices in the g sector.

There are several other things to note about this figure. When prices are flexible in sector s – the far left of each panel, there is overall inflation of one percent (panel A), sector g prices are nearly constant (panel B), relative price distortions are nearly eliminated (panel C) and the markup is nearly identical across sectors at its flexible price level (panel D). In this case, it is optimal to place the entire burden of price adjustment on the flexible price sector. I use the qualifier "nearly" because, as explained in King and Wolman [1999], a very small positive trend in prices allows the monetary authority to increase welfare by decreasing the markup in sector g.

Another interesting feature of Figure 5 is that when price stickiness does not vary across sectors (expected duration equals two), the optimal PCE inflation rate is not zero but -0.4%. This is because of the higher expenditure share on services: with the same degree of price stickiness in both sectors, it is optimal to impose more of the burden of adjustment on the smaller sector. In this case, sector g prices must fall more than sector s prices rise, as illustrated in panel B at the point where expected duration equals two.

Panel C and D of Figure 5 shows non-monotonicity in the relative price distortion and markup for sector s. If money growth were held fixed while the duration of prices increased, then these relationships would be monotonic. However, as can be seen from panel A, inflation (and hence money growth) varies with the expected duration, creating the possibility of nonmonotonicity.

For Figure 6, I fix the rate of productivity growth in the g sector at its calibrated value and vary the rate of productivity growth in the s sector. The focal point in this figure is a growth rate of 0.00915; at that point productivity growth is identical in the two sectors. Then there is no trend in relative prices (panel B), and the optimal steady state involves approximately zero inflation (panel A).[11] In Figure 6, the sector s relationships have less curvature than those for sector g. This is because less price stickiness and a smaller expenditure share for sector g makes it optimal for nominal price changes in that sector to bear most of the burden of accommodating the different degrees of relative price change implied by different relative productivity growth rates. Thus, in panel B the sectoral price change relationship for sector g is steeper than that for sector s.

7. Conclusion

Given the relative price trend across sectors, the optimal rate of inflation in our model is driven by heterogeneity in price stickiness. Relative degrees of price stickiness are likely robust determinants of the optimal rate of inflation in the presence of relative price trends. However, the analysis here raises the issue of other ways in which sectoral heterogeneity might interact with monetary policy. I use this concluding section to suggest a few possibilities for further research along these lines.

I have taken as given the degree of price stickiness in the two sectors, and held it fixed with respect to policy changes. Presumably the frequency of price adjustment responds to the average rate of sectoral price change. It is difficult to find such a relationship in the Bils and Klenow data, but it may be obscured by other sectoral heterogeneity, say in the form of idiosyncratic productivity or cost shocks. It would be interesting to include such firm-level heterogeneity and

[11] Although it is feasible to eliminate the relative price distortions in both sectors when the productivity growth rates are identical, it is optimal to have a bit of inflation in order to decrease the markup. This is another version of the effect described by King and Wolman (1999).

extend the analysis to a state-dependent pricing framework; there the frequency of price adjustment would naturally respond to the average rate of price change.

As mentioned above, money demand may interact with sectoral heterogeneity. Here there are no transactions frictions motivating money demand. Of course those frictions exist. Furthermore, they may differ systematically across sectors, and they may be correlated with price rigidities. It is unlikely that such heterogeneity would by itself overturn the Friedman rule, but it might have other interesting implications.

I analyzed a stylized model containing two types of consumption differing only in the stickiness of prices within the sectors and in productivity growth. There are other dimensions of heterogeneity which ought to be modeled, and have to some extent been modeled by others: durability (e.g. Erceg and Levin [2006]), money demand (just mentioned), intermediate input shares (Huang and Liu [2005]), the extent of competition, etc.

Although expenditure shares appear to have stabilized, there was a long run change in expenditure shares at least until the mid-1990s. Ideally the model could help us to understand shifting expenditure shares over time. A paper along these lines is Greenwood and Uysal [2004], although they do not consider monetary policy. A model with shifting expenditure shares might be an interesting laboratory for studying changes in the composition of means of payment.

References

[1] Aoki, Kosuke, "Optimal Monetary Policy Response to Relative Price Changes," *Journal of Monetary Economics* 48 (2001), 55-80.

[2] Benigno, Pierpaolo, "Optimal Monetary Policy in a Currency Area," *Journal of International Economics* 63 (2004), 293-320.

[3] Bils, Mark and Pete Klenow, "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy*, October 2004.

[4] Bosworth, Barry P., "Economic Policy" in Joseph A. Pechman ed., *Setting National Priorities; Agenda for the 1980s.* (1980) Washington DC: Brookings Institution.

[5] Carlstrom, Charles T., Timothy S. Fuerst and Fabio Ghironi, "Does It Matter (for Equilibrium Determinacy) What Price Index the Central Bank Targets?" *Journal of Economic Theory* 128 (May 2006): 214-231.

[6] Ding, Fan and Alexander L. Wolman, "Inflation and Changing Expenditure Shares," *Federal Reserve Bank of Richmond Economic Quarterly* 91 (2005).

[7] Erceg, Christopher, Dale Henderson and Andrew Levin, "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, vol. 46 (October 2000), pp. 281-313

[8] Erceg, Christopher and Andrew Levin, "Optimal Monetary Policy with Durable Consumption Goods." *Journal of Monetary Economics* 53 (October 2006), pp. 1341-1359

[9] Greenwood, Jeremy and Gokce Uysal, "New Goods and the Transition to a New Economy," NBER Working Paper 10793, September 2004.

[10] Huang, Kevin, and Zheng Liu, "Inflation Targeting: What Inflation Rate to Target?" *Journal of Monetary Economics* 52(8), November 2005, pp. 1435-1462.

[11] King, Robert G. and Alexander L. Wolman, "Inflation Targeting in a St. Louis Model of the 21st Century," *Federal Reserve Bank of St. Louis Review*, May/June, 1996

[12] King, Robert G. and Alexander L. Wolman, "What Should the Monetary Authority Do When Prices are Sticky?" in John B. Taylor ed. *Monetary Policy Rules*, 1999.

[13] Loyo, Eduardo, "Imaginary Money Against Sticky Relative Prices," *European Economic Review* 46 (2002), 1073-1092.

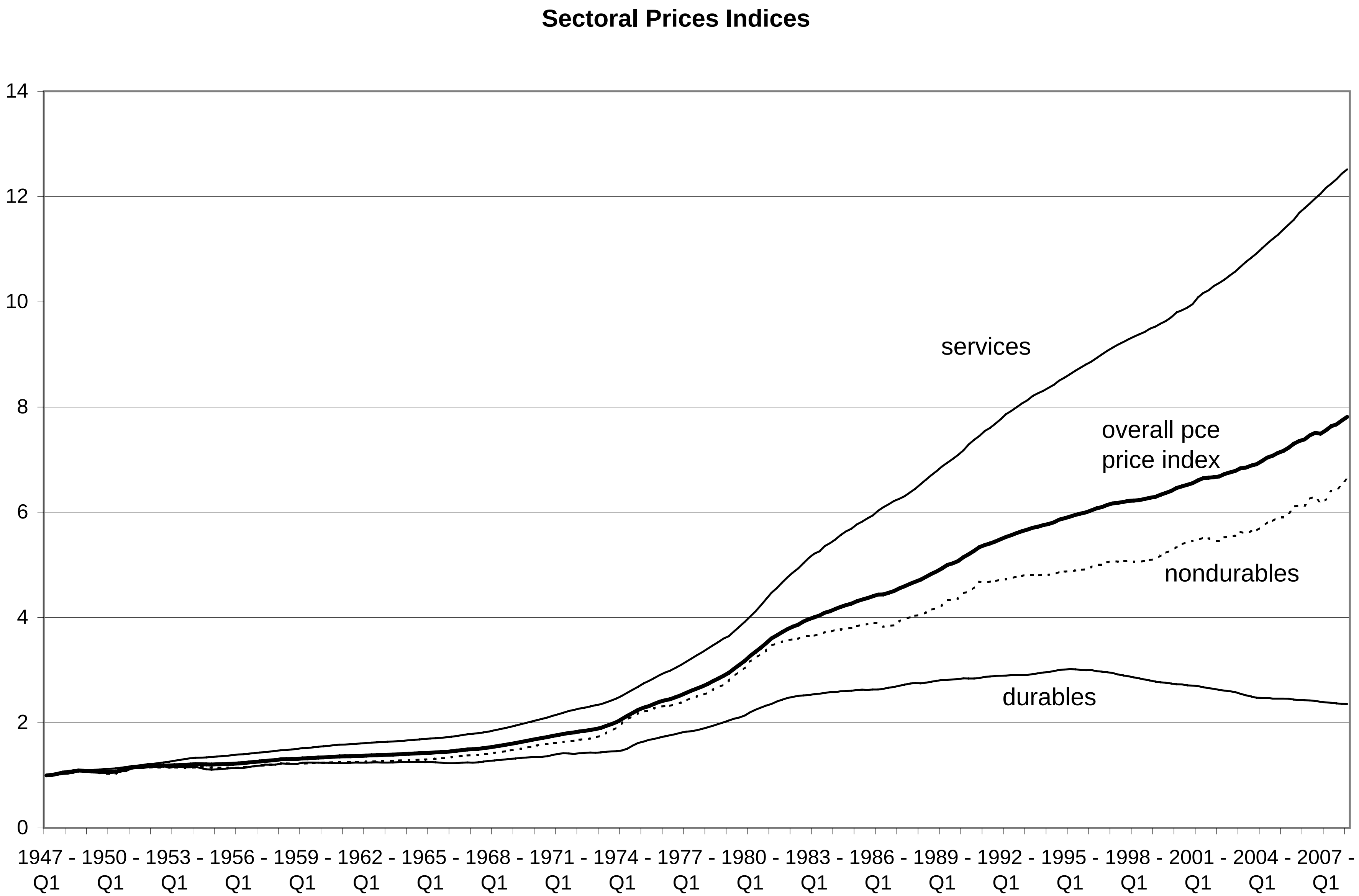
[14] Ngai, L. Rachel, and Christopher Pissarides, "Structural Change in a Multi-Sector Model of Growth," manuscript, LSE.

[15] Shirota, Toyoichiro, "Optimal Trend Inflation and Monetary Policy under Trending Relative Prices." Bank of Japan Working Paper Series No. 07-E-1, January 2007.

[16] Wolman, Alexander L., "Nominal Frictions, Relative Price Adjustment, and the Limits to Monetary Policy." Federal Reserve Bank of Richmond *Economic Quarterly*, forthcoming.

[17] Wolman, Alexander L., "Sticky Prices, Marginal Cost and the Behavior of Inflation," *Federal Reserve Bank of Richmond Economic Quarterly* 85 (Fall 2000) , 29-48.

Figure 1.



Sectoral Prices Indices
14
12
10
8
6
4
2
0
services
overall pce
price index
nondurables
durables
1947 - Q1
1950 - Q1
1953 - Q1
1956 - Q1
1959 - Q1
1962 - Q1
1965 - Q1
1968 - Q1
1971 - Q1
1974 - Q1
1977 - Q1
1980 - Q1
1983 - Q1
1986 - Q1
1989 - Q1
1992 - Q1
1995 - Q1
1998 - Q1
2001 - Q1
2004 - Q1
2007 - Q1

Figure 2.

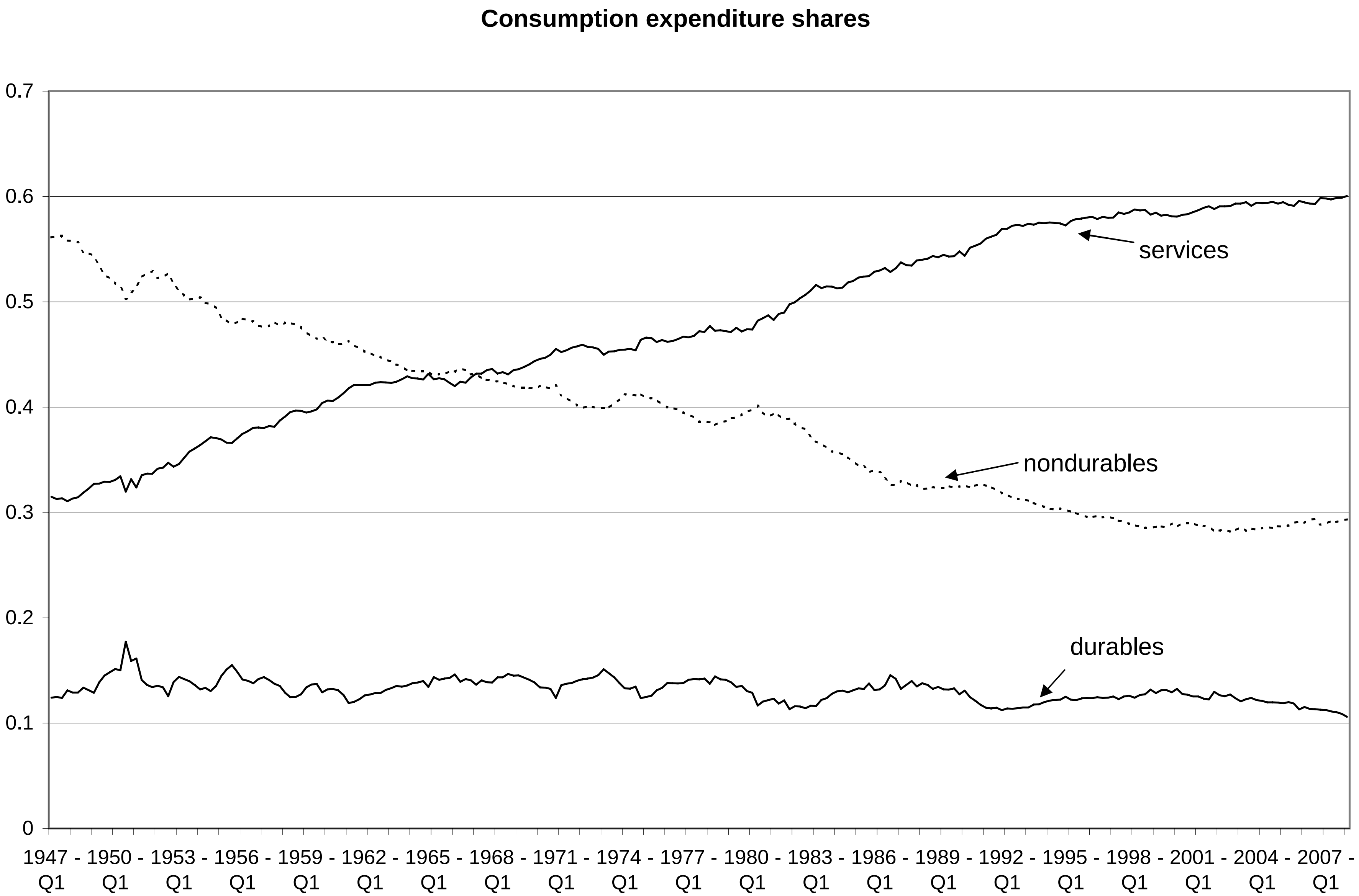

Consumption expenditure shares
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
services
nondurables
durables
1947 - Q1
1950 - Q1
1953 - Q1
1956 - Q1
1959 - Q1
1962 - Q1
1965 - Q1
1968 - Q1
1971 - Q1
1974 - Q1
1977 - Q1
1980 - Q1
1983 - Q1
1986 - Q1
1989 - Q1
1992 - Q1
1995 - Q1
1998 - Q1
2001 - Q1
2004 - Q1
2007 - Q1

Figure 3.

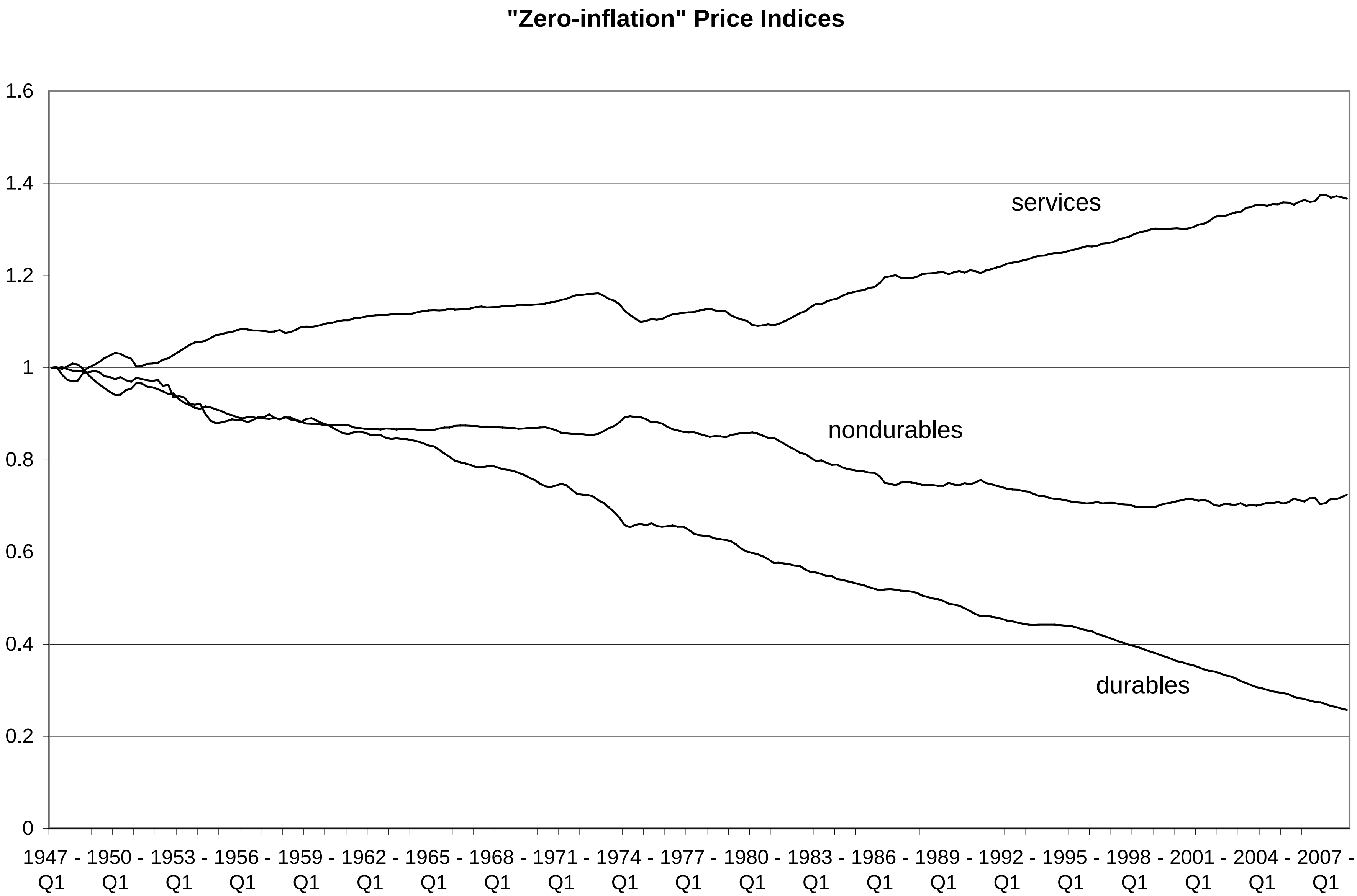

"Zero-inflation" Price Indices
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
services
nondurables
durables
1947 - Q1
1950 - Q1
1953 - Q1
1956 - Q1
1959 - Q1
1962 - Q1
1965 - Q1
1968 - Q1
1971 - Q1
1974 - Q1
1977 - Q1
1980 - Q1
1983 - Q1
1986 - Q1
1989 - Q1
1992 - Q1
1995 - Q1
1998 - Q1
2001 - Q1
2004 - Q1
2007 - Q1

Figure 4. Inflation and Welfare
solid line=g, dashed=s

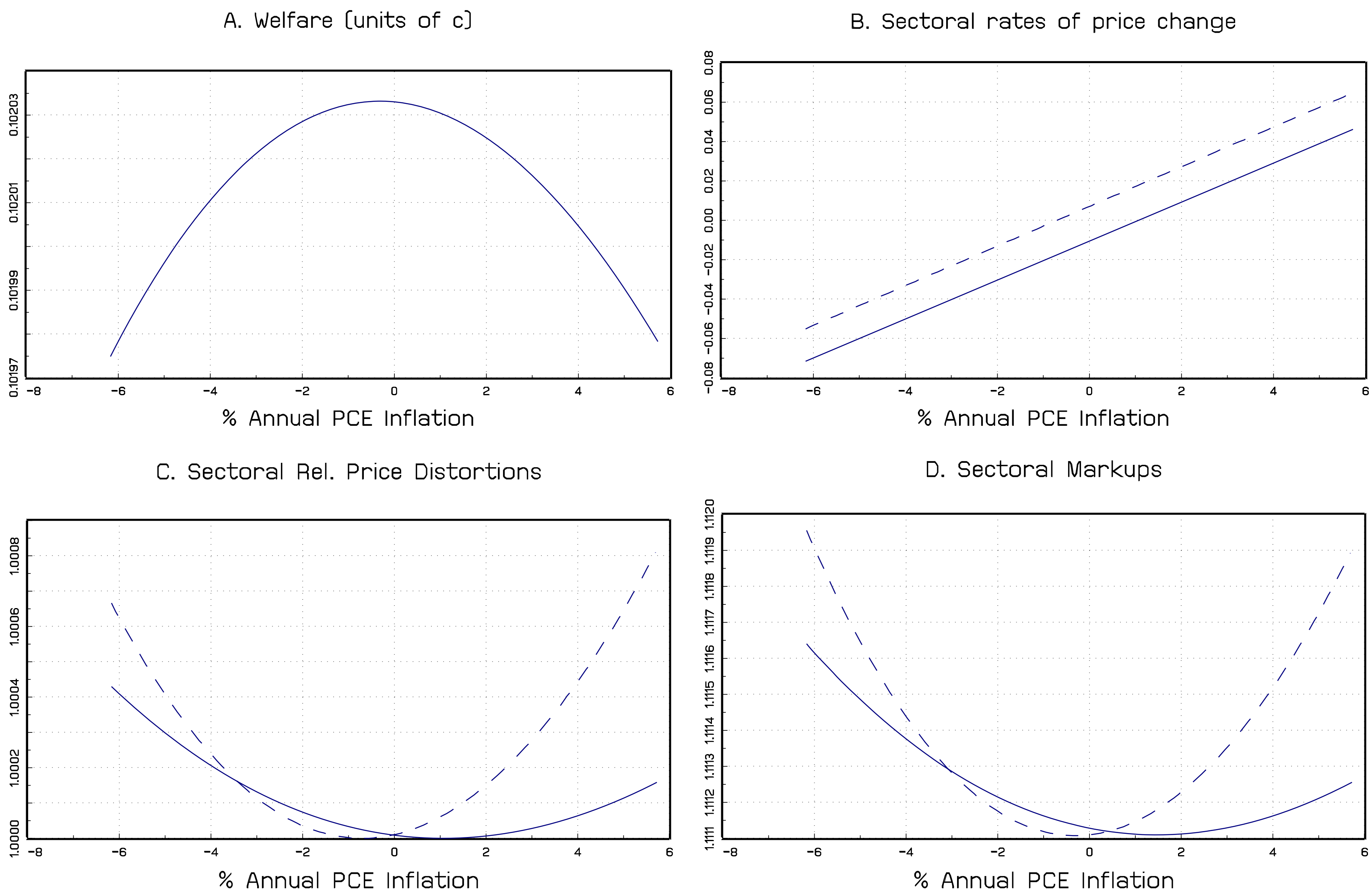


Figure 5. Sensitivity of optimal inflation to expected duration of prices in sector s

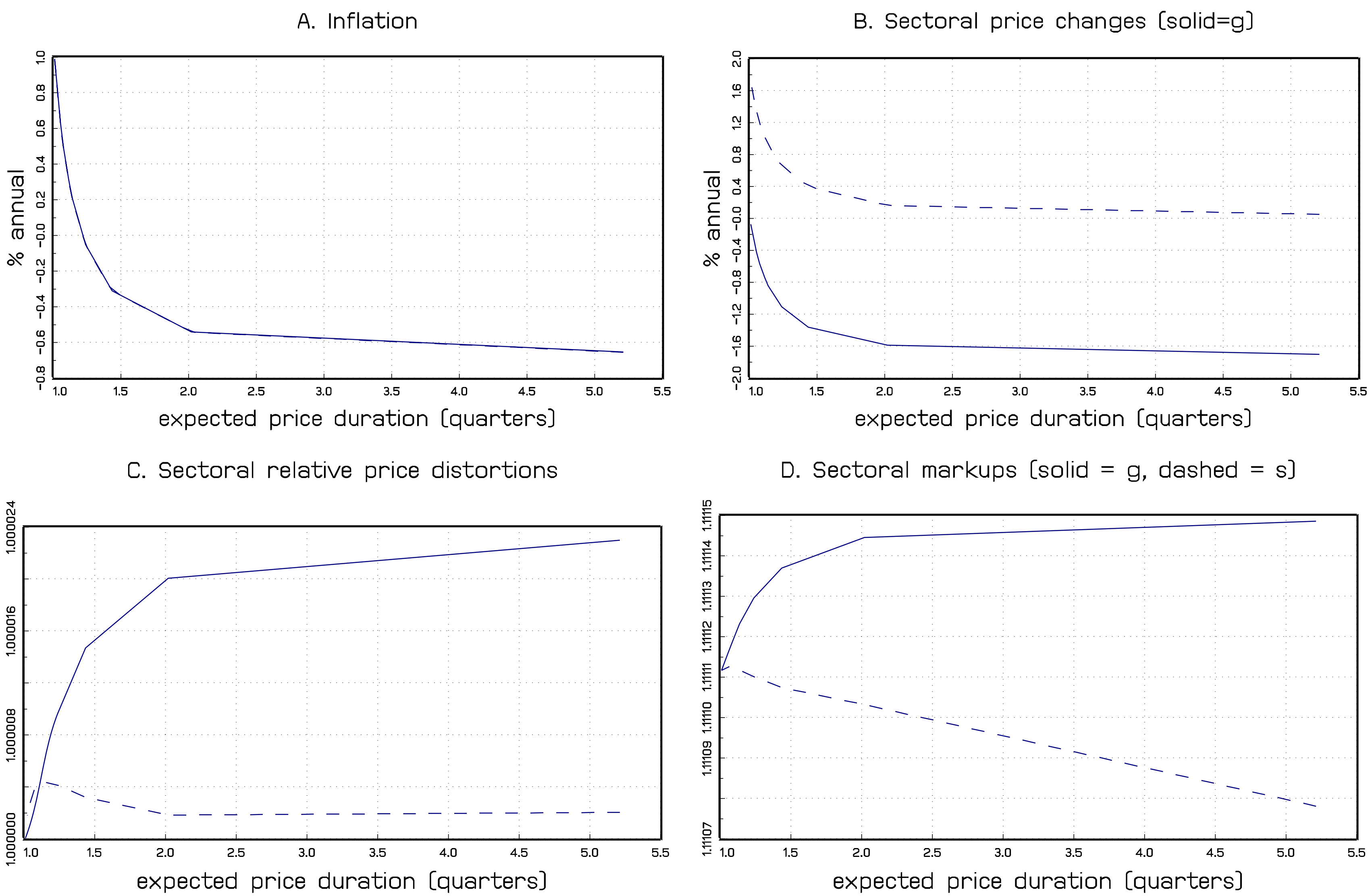


Figure 6. Sensitivity of optimal inflation to productivity growth in sector s