Working Paper Series

# Unemployment Insurance with a Hidden Labor Market

WP 09-09

Fernando Álvarez-Parra
Banco Central de Venezuela

Juan M. Sanchez
Federal Reserve Bank of Richmond



THE FEDERAL RESERVE BANK OF RICHMOND
RICHMOND ▪ BALTIMORE ▪ CHARLOTTE

# Unemployment Insurance with a Hidden Labor Market*

Fernando Álvarez-Parra† Juan M. Sánchez‡

June 2009

Working Paper No. 09-09

**Abstract**

This paper considers the problem of optimal unemployment insurance (UI) in a repeated moral hazard framework. Unlike existing literature, unemployed individuals can secretly participate in a *hidden labor market*. This extension modifies the standard problem in three dimensions. First, it imposes an endogenous lower bound for the lifetime utility that a contract can deliver. Second, it breaks the identity between unemployment payments and consumption. And third, it hardens the encouragement of search effort. The optimal unemployment insurance system in an economy with a hidden labor market is simple, with an initial phase in which payments are relatively flat during unemployment and with no payments for long-term unemployed individuals. This scheme differs substantially from the one prescribed without a hidden labor market and resembles unemployment protection programs in many countries.

***JEL Classification***: D82, H55, I38, J65
***Keywords***: Unemployment Insurance, Hidden Labor Markets, Recursive Contracts

*This paper was written when we were graduate students at the University of Rochester. We gratefully acknowledge Árpád Ábrahám for constant encouragement and valuable advice. In addition, we are grateful to Mark Aguiar for enhancing the quality of the paper with detailed comments. We also thank Fernando Alvarez, Rachana Bhatt, Harold Cole, Jeremy Greenwood, William Hawkins, Cagatay Kayi, Dirk Krueger, Luis Marques, Juan Pablo Nicolini, Jose-Victor Rios-Rull, Anne Stilwell, Jesús Fernández-Villaverde, Gianluca Violante and other seminar participants at the University of Rochester, Torcuato Di Tella, Wharton, European Workshop in Macroeconomics, and Midwest Macro Meetings. We thank Elaine Mandaleris-Preddy for writing guidance. All remaining errors are our own. The views expressed in this paper are those of the authors, and do not necessarily reflect those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

†Banco Central de Venezuela. Email: fealvare@bcv.org.ve.

‡Federal Reserve Bank of Richmond. Email: juan.m.sanchez@rich.frb.org.

# 1 Introduction

Unemployed individuals face a trade-off between search effort and leisure. Since effort is unobservable, the design of unemployment protection programs must balance insurance and incentives. This moral hazard problem has been the object of study since the pioneering work of Shavell and Weiss (1979). In this literature, they assume that employment status is observable and hence individuals cannot defraud the system by working and asking for unemployment payments. But what if there is a shadow economy that allows unemployed individuals to secretly work and, simultaneously, ask for unemployment payments? This paper explores the implications of a hidden labor market in the design of optimal unemployment insurance (UI). There are at least three facts suggesting that incorporating a hidden labor market in the study of optimal unemployment insurance might be important:[1]

- The hidden labor market or the shadow economy is important in many countries, accounting for 13-30 percent of GDP in industrialized economies to 39-76 percent for African countries (Schneider and Enste, 2000).
- In economies with a sizeable shadow economy, unemployment is a likely event. On average, unemployment rates range from an average of 6.8 in industrialized economies to 11.6 for African countries (ILO, 2004).
- While coverage, replacement rate, and benefit duration are different across countries, most protect the individuals against unemployment risk.[2] Replacement rates are around 45-75 percent over 1-2 years (Vodopivec and Raju, 2002).

[1]The informal sector is interpreted as a hidden labor market in which the individual can always find a job but his productivity is lower than in the formal sector.

[2]Also, the number of countries providing unemployment insurance programs increased from 21 in 1940 to 68 in 1997 (based on Social Security Programs Throughout the World, Social Security Administration, U.S., 1997).

Incorporating a hidden labor market modifies the standard unemployment insurance problem in three dimensions. First, it imposes an endogenous lower bound for the lifetime utility that a contract can deliver. Second, it breaks the identity between unemployment payments and consumption, since unemployed individuals may also obtain resources by working in the informal sector. And third, when more participation in the hidden labor market increases the cost of search effort, the presence of a hidden labor market makes it harder to encourage the job search effort.

Important insights can be obtained in a simple case in which more participation in the hidden labor market does not increase the cost of search effort (linear effort-cost function). Initially, consumption is strictly decreasing during unemployment and individuals do not participate in the hidden labor market. A decreasing path of payments provides incentives for a search effort because unemployed individuals search harder in order to increase expected future income. Since payments are high enough, obtaining extra resources from the hidden labor market is not worth the effort. Together with the drop in payments, there is also a reduction in the continuation utility the contract promises to deliver. For sufficiently large unemployment spells, those promised utilities eventually reach a critical point–the lower bound. At that moment alone, the optimal contract prescribes positive participation in the hidden labor market along with zero unemployment payments. With the jump in participation, the individual smooths out the abrupt decline in payment and hence the consumption path remains smooth. For the more general case, in which more participation in the hidden labor market increases the cost of search effort (non-linear effort-cost function), the numerical solution of the problem is used to characterize the optimal contract. In particular, the model is calibrated to match some moments from the Spanish economy. In this framework, a new intermediate phase arises as a consequence of the effect of participation in the hidden labor market on the cost of search effort. In this phase, the unemployment payment profile is flattened. To the best of our knowledge, this is the first paper justifying a *flattened* payment profile with a subsequent *abrupt decline*. The comparison with alternative environments sheds light on the causes of the flattening and the abrupt decline. The existence of an alternative source of income during unemployment is responsible for the optimality of eliminating unemployment payments for a sufficiently long unemployment spell. Otherwise, long unemployment spells are not serious threats that encourage search effort. The non-observability of the participation in this alternative labor market is responsible for the flattening in the payment schedule. With a steeper profile of payments, unemployed individuals would prefer to deviate from the contract and participate in the hidden labor market.

Unemployment insurance systems have been studied from two different perspectives. One approach uses quantitative general equilibrium models to study the influence of unemployment insurance systems on macroeconomic variables and welfare (Hansen and Imrohoroglu, 1992; Wang and Williamson, 1996). The other approach, followed in this paper, uses contract theory in a partial equilibrium environment to study the optimal design of unemployment insurance. In a repeated moral hazard framework, early contributions find that payments must decrease over the unemployment spell in order to encourage search effort (Shavell and Weiss, 1979; Hopenhayn and Nicolini, 1997). A controversial result from this early literature is that the optimal contract leads unemployed individuals toward "immiserization".[3] To prevent this result, Pavoni (2007) imposes an arbitrary lower bound for the promised utility and characterizes the resulting optimal unemployment insurance.[4] Our work is related to

[3]More precisely, if the individuals' utility function is unbounded below, then efficiency requires that the individuals' expected discounted utility falls, with positive probability, below any arbitrary negative level (Pavoni, 2007). In this environment, it means that if an individual is unemployed for a long enough time, utility would fall below any bound.

[4]There are several related environments where "immiserization" does not hold. For instance, see Wang (1995) and Phelan (1995).

this idea because lifetime utility cannot fall below a lower bound. However, our paper differs from Pavoni (2007) in crucial ways. It justifies the existence of a lower bound in lifetime utility by including a hidden labor market where individuals can guarantee themselves a minimum consumption. This is important because the lower bound here is connected to the structure of the economy. More importantly, the dynamics of unemployment payments is substantially modified by the presence of a hidden labor market, even before the lower bound for promised utility is reached.

The policy implications of a hidden labor market have been studied in different environments and with different goals. Among others, labor regulations, wage controls, migration policies, and taxation are analyzed in frameworks with hidden labor markets by Banerjee (1983), Fortin, Marceau, and Savard (1997), and Johnson, Kaufmann, and Zoido-Lobaton (1998). Surprisingly, there is no paper in this strand of literature considering the optimal design of unemployment insurance. In that sense, the closest study is Hopenhayn and Nicolini (2005). They focus on individuals' incentives to accept bad jobs when layoffs and quits are undistinguishable. In particular, their interest is about individuals accepting and then quitting those jobs just to upgrade their unemployment insurance benefits. Instead, this paper studies the possibility that those jobs are non-observable, which implies that individuals could accept them and, simultaneously, ask for unemployment payments.

Section 2 introduces the model. Section 3 characterizes, qualitatively, the optimal unemployment insurance with a hidden labor market. Section 4 studies the quantitative implications of the model calibrated to Spain. Section 5 concludes. All the formal results are explained in the main text, but the complete proofs are delegated to the Online Appendix.

# 2 Model

## 2.1 Environment

The economy is populated by a continuum of infinitely lived risk averse *individuals*. At each period, an individual's labor status is denoted by $m_t$. The variable $m_t \in \{u, e\}$, where $u$ stands for unemployment and $e$ for employment. The employment history of an individual up to time $t$ is represented by $h^t = (m_0, ..., m_t)$.

In contrast to previous studies, this environment includes a secondary or hidden labor market (also referred to as informal sector or shadow economy) in which individuals obtain a wage, $\varpi$, for unit of effort, $s$. Hereafter, $s$ will be referred to as participation in the hidden labor market.

Individuals employed in the formal sector, *employed individuals*, exert a constant work effort $o$, have productivity $\omega$, and receive (after tax) wages $w$. Employment relationships end exogenously with probability $\delta$. By assumption, participation in the hidden labor market during employment and on-the-job search are ruled out. Additionally, productivity in the hidden labor market is lower than in the formal sector, $\varpi < \omega$.

Unemployed individuals search for a job and can, simultaneously, participate in the hidden labor market. When exerting search effort $a$, they find a job next period with probability $p(a)$. The total effort while unemployed, $x$, is restricted to lie in the compact set $[0, \vartheta]$, where $\vartheta$ is an arbitrary constant. The function $p$ is strictly increasing, strictly concave, twice differentiable and satisfies standard Inada conditions.

The instantaneous felicity function is separable in consumption and total effort

$$u(c) - v(x), \tag{1}$$

where $c$ is the individual's consumption. The function $u$ reflects the utility from consumption. It is strictly increasing and bounded above by $\overline{u}$. It is also strictly concave, twice differentiable, and satisfies Inada conditions. Similarly, the function $v$ measures the cost of effort exerted either searching for a job, participating in the hidden labor market or working in the formal sector. This function is convex, strictly increasing, twice differentiable, and satisfies $v(0) = 0$. Notice that total effort, $x$, equals $o$ if the individual is employed and $s + a$ otherwise. Finally, individuals can neither save nor borrow and they discount future utility with the factor $\beta \in (0, 1)$.

There is also a risk-neutral *planner* who provides unemployment insurance and affects individuals' earnings by levying after-reemployment taxes. The planner cannot observe either hidden labor market participation, $s$, or job search effort, $a$. The planner can borrow and save at a constant interest rate $r = 1/\beta - 1$.

## 2.2 Sequential formulation

The planner chooses unemployment payments, after reemployment wages, search effort, and hidden labor market participation. The collection of those decisions for each possible history node constitutes a contract.

**Definition 1** *A* ***contract*** *or* ***unemployment insurance system****, $\mathcal{W}$, is a collection of functions specifying unemployment payments, $b$, wages after reemployment, $w$, search effort, $a$, and hidden labor market participation, $s$, for each possible history, $h^t$, at each period; i.e., $\mathcal{W} = \{b_t(h^t), w_t(h^t), a_t(h^t), s_t(h^t), \forall h^t, t\}$.*

Notice that a contract can be divided into two components: those representing the planner's direct instruments, $\mathcal{I} = \{b_t(h^t), w_t(h^t), \forall h^t, t\}$, and those representing the planner's prescriptions for non-observable individuals' actions, $\mathcal{P} = \{a_t(h^t), s_t(h^t), \forall h^t, t\}$. Then, a contract is feasible if it provides non-negative consumption and effort is smaller than the upper bound; i.e., $\mathcal{W}$ is feasible if $(\mathcal{P}, \mathcal{I}) \in \mathbf{A} \times \mathbf{T}$, where

$$\begin{aligned}\mathbf{A} = \{\mathcal{P} :\ & \text{for each } h^t \text{ and each } t,\ (a_t(h^t), s_t(h^t)) \in [0, \vartheta]^2, a_t(h^t) + s_t(h^t) \leq \vartheta \\ & \text{if } m_t = u, \text{ and } a_t(h^t) = s_t(h^t) = 0 \text{ if } m_t = e\},\end{aligned}$$

$$\begin{aligned}\mathbf{T} = \{\mathcal{I} : b_t(h^t) \geq 0 \text{ for each } h^t, & \text{ with equality if } m_t = e, \text{ and} \\ & w_t(h^t) \geq 0 \text{ with equality if } m_t = u\}.\end{aligned} \tag{2}$$

Thus, each contract prescribes sequences for consumption, $c$, total effort, $x$, and one-period planner's payoffs, $y$, for each employment state. This can be described with the notation introduced up to here as follows.

$$\begin{aligned}c_t(h^t) &= \begin{cases} b_t(h^t) + s_t(h^t)\varpi & \text{if } m_t = u \\ w_t(h^t) & \text{if } m_t = e \end{cases} \\ x_t(h^t) &= \begin{cases} a_t(h^t) + s_t(h^t) & \text{if } m_t = u \\ o & \text{if } m_t = e \end{cases} \\ y_t(h^t) &= \begin{cases} -b_t(h^t) & \text{if } m_t = u \\ \omega - w_t(h^t) & \text{if } m_t = e. \end{cases}\end{aligned} \tag{3}$$

The lifetime utility that an individual obtains from a contract $\mathcal{W}$, from a particular history $h^t$ on, is referred to as $\mathcal{U}_t(\mathcal{W}; h^t)$. Likewise, $\mathcal{V}_t(\mathcal{W}; h^t)$ represents the profits that the planner obtains from this pair $(\mathcal{W}; h^t)$. Formally, these values are defined by

$$\mathcal{U}_t(\mathcal{W}; h^t) \equiv E_t\left[\sum_{n=t}^{\infty} \beta^{n-t}(u(c_n(h^n)) - v(x_n(h^n)))|\mathcal{W}, h^t\right], \tag{4}$$

and

$$\mathcal{V}_t(\mathcal{W}; h^t) \equiv E_t\left[\sum_{n=t}^{\infty} \beta^{n-t} y_n(h^n)|\mathcal{W}, h^t\right]. \tag{5}$$

Since search effort and hidden labor market participation are not observable, the unemployment insurance scheme has to be *incentive compatible*. This means that the levels of search effort and hidden labor market participation prescribed by the contract must be consistent with the individuals' choices. The next definition states this formally.

**Definition 2** *A contract $\mathcal{W} = (\mathcal{P}, \mathcal{I})$ is* ***incentive compatible*** *if for each history and each period, there is not an alternative feasible action $\widetilde{\mathcal{P}}$ providing a higher lifetime utility for the individual than the one induced by the contract; i.e., for each $h^t$ and $t$,*

$$\mathcal{U}_t((\mathcal{P}, \mathcal{I}); h^t) \geq \mathcal{U}_t((\widetilde{\mathcal{P}}, \mathcal{I}); h^t) \text{ for each } \widetilde{\mathcal{P}} \in \mathbf{A}. \tag{6}$$

In an unemployment insurance model without hidden labor markets, Pavoni (2007) shows that the optimal feasible and incentive compatible contract "implies a weaker form of what is known as the "immiserization" result: if the individuals' utility function is unbounded below, then efficiency requires that individuals' expected discounted utility falls, with positive probability, below any arbitrary negative level."[5] In the environment presented in this paper, it is clear that the planner cannot punish the individual with promised utility below the critical value $\underline{U}$, where

$$\underline{U} \equiv \frac{u(s^*\varpi) - v(s^*)}{1-\beta}, \tag{7}$$

and $s^*$ solves

$$u'(s^*\varpi)\varpi = v'(s^*). \tag{8}$$

The lifetime utility $\underline{U}$, referred to as the *lower bound* for promised utility hereafter, is the value that an individual attains with no job search and zero unemployment payments at any period and with participation in the hidden labor market chosen optimally. Lemma 1 formally states this result.

[5] See proposition 3 in Pavoni (2007). Green (1987) and Thomas and Worral (1990) in repeated unobservable endowments economies prove a similar but stronger result.

**Lemma 1 (No "immiserization")** *In the optimal contract there is not "immiserization." In particular, the promised utilities provided by the constrained efficient contract* $\mathcal{W}^*$ *are bounded below by* $\underline{U}$.

The proof of this result is very intuitive: the hidden labor market provides a protection for the individuals that prevents "immiserization." Whenever the planner pretends to deliver a promised utility below $\underline{U}$, the individual deviates and obtains at least $\underline{U}$.[6]

A direct implication of the last result is that the planner's problem (formally stated later) is not well defined for lifetime utilities smaller than the lower bound, $U < \underline{U}$, because the set of incentive compatible contracts is empty. Similarly, it is useful to recall that $u$ is bounded above $\overline{u}$. As a consequence, the lifetime utilities the contract can deliver are bounded above by $\overline{U} \equiv \frac{\overline{u}}{1-\beta}$. Thus, the set of lifetime utilities that can be delivered by the contract can be characterized.

**Lemma 2** *Define* $\mathbf{C}(U; h^0)$ *as the set of incentive compatible contracts providing promised utility greater than or equal to* $U$ *given initial state* $h^0$*; i.e.,*

$$\mathbf{C}(U; h^0) \equiv \{\mathcal{W} : \mathcal{U}_0((\mathcal{P}, \mathcal{I}); h^0) \geq U\}. \tag{9}$$

*Then, for any* $\overline{U} > U \geq \underline{U}$*, the set* $\mathbf{C}(U; h^0)$ *is not empty.*

To show that the set of feasible and incentive-compatible contracts is not empty for a given promised utility, it is enough to find one contract satisfying those conditions. This contract is actually simple. It provides the same one-period utility across time and histories, zero search effort, the optimal participation in the hidden labor market (given the rest of the

[6]Notice that the assumption that payments are non-negative is crucial to prevent "immiserization" in our environment. Negative unemployment payments could be understood in this environment as taxes to the hidden labor market activity. This possibility is ruled out because this kind of taxes are not used in actual economies.

contract), and the level of payments and wages that deliver the promised utility. Thus, the set of promised utilities that the contract can deliver is defined by

$$\mathbf{P}_u = \mathbf{P}_e = \mathbf{P} \equiv \{U \in \mathbb{R} : \overline{U} > U \geq \underline{U}\}. \tag{10}$$

Now, the sequential problem can be stated. The profits from a contract that provides lifetime utility $U$ to an individual whose initial labor status equals $m_0$ are

$$V^{m_0}(U) = \max_{\mathcal{W} \in \mathbf{F}} \mathcal{V}_0(\mathcal{W}; h^0) \tag{11}$$

$$s.t$$

$$\mathcal{W} \in \mathbf{C}(U, m_0), \tag{12}$$

$$\mathcal{U}_0(\mathcal{W}; h^0) \geq U, \tag{13}$$

$$\mathcal{U}_t(\mathcal{W}; h^t) \in \mathbf{P}, \forall h^t, t. \tag{14}$$

The constraint (12) guarantees that the individual does not want to choose different search effort and participation in the hidden labor market than those prescribed by the contract. The constraint (13) guarantees that the individual's lifetime utility is higher or equal to $U$. The solution to the problem (11)-(14) is called *constrained efficient contract* or *optimal unemployment insurance scheme*. Hereafter, this contract is studied using the recursive representation of this problem.

## 2.3 Recursive representation

Because of its dependence on history, the problem (11)-(14) is not tractable for quantitative purposes. Fortunately, following the lines of Spear and Srivastava (1987) and Abreu, Pearce, and Stacchetti (1990), the problem has a recursive representation in which the history is completely encoded in two single variables, namely, promised utility and labor status. As implied by lemma 1 and 2, the planner's problem is well defined if and only if promised utility is not smaller than $\underline{U}$. Therefore, the recursive representation must explicitly impose that promised utility belongs to $\mathbf{P}$. Before presenting the recursive planner's problem, the definition of a *recursive contract* is formally stated.

**Definition 3** *A* ***recursive contract****, or* ***unemployment insurance system****, $\mathcal{W}^R$, is a collection of functions specifying unemployment payments, $b$, wages after reemployment, $w$, search effort, $a$, hidden labor market participation, $s$, and promised utility in any next period state $m$, $U^m$, for each $U$ in $\mathbf{P}$; i.e., $\mathcal{W}^R = \{b(U), w(U), a(U), s(U), U^u(U), U^e(U)\}$.*

A recursive contract can also be divided into a planner's prescriptions for individuals' actions, $\mathcal{P}^R = \{a(U), s(U)\}$, and direct instruments, $\mathcal{I}^R = \{b(U), w(U), U^u(U), U^e(U)\}$. Let $\mathbf{E}$ be the set of one-period feasible actions and $\mathbf{M}$ the set of one-period best response of individuals, taking as given unemployment payments and promised utility; i.e.,

$$\mathbf{E} \equiv \{(a, s) : (a, s) \in [0, \vartheta]^2\},$$

$$\mathbf{M}(b, U^e, U^u) \equiv \{\text{argmax}_{(a,s)\in\mathbf{E}}\ u(b + s\varpi) - v(a + s) + \beta[p(a)U^e + (1 - p(a))U^u]\}.$$

Then, the planner's problem is represented by the following functional equations system:

$$V^u(U) = \max_{b,a,s,U^u,U^e} -b + \beta[p(a)V^e(U^e) + (1 - p(a))V^u(U^u)], \tag{15}$$

$s.t.$

$$u(b + s\varpi) - v(s + a) + \beta[p(a)U^e + (1 - p(a))U^u] - U = 0, \tag{16}$$

$$(a, s) \in \mathbf{M}(b, U^e, U^u), \tag{17}$$

$$(U^u, U^e) \in \mathbf{P}^2, \tag{18}$$

where

$$
\begin{aligned}
V^e(U) = \max_{\tilde{U}^u, \tilde{U}^e, w} \quad & \omega - w + \beta[\delta V^u(\tilde{U}^u) + (1-\delta) V^e(\tilde{U}^e)] \\
s.t. \quad & \\
& u(w) - v(o) + \beta[(1-\delta)\tilde{U}^e + \delta \tilde{U}^u] - U = 0, \\
& (\tilde{U}^u, \tilde{U}^e) \in \mathbf{P}^2.
\end{aligned}
$$

Equation (16) is the promise-keeping constraint in recursive form, and (17) is the incentive-compatibility constraint. Finally, (18) restricts promised utilities to the set $\mathbf{P}$. This constraint is referred to as the *feasibility of promised utility constraint* hereafter.

The inconvenience with the previous representation is that constraint (17) is a complicated object that makes the problem non-tractable. A common strategy is the so-called *first order condition approach*. To describe this strategy, consider an unemployed individual who takes as given $(b, U^u, U^e)$ and chooses $(a, s) \in \mathbf{E}$ to maximize utility.[7] For such a problem the first order conditions are:

$$\beta p'(a)[U^e - U^u] - v'(s+a) = 0, \tag{19}$$

$$-u'(b + s\varpi)\varpi + v'(s+a) \geq 0. \tag{20}$$

Define $\mathbf{M}^{foc}$ as the set of all $(a, s) \in \mathbf{E}$ such that equations (19)-(20) are satisfied. The first order condition approach basically replaces constraint (17) in the recursive problem (15)-(18) by the condition that $(a, s) \in \mathbf{M}^{foc}$.[8]

[7]Under the assumption that $p$ satisfies Inada conditions and provided that in equilibrium $U^e > U^u$, the non-negativity constraint for $a$ is never binding. Likewise, $\vartheta$ can be set big enough to avoid $a + s = \vartheta$. Hence, the only relevant non-negativity constraint is the one for participation in the hidden labor market.

[8]Since $\mathbf{M} \subseteq \mathbf{M}^{foc}$, the solution to the problem using the first order condition approach may differ from the solution to the original problem. A recent example is provided in Kocherlakota (2004). In our environment, given $(b, U^u, U^e)$, the individual's problem is convex. This implies that the first order conditions are both necessary and sufficient and, hence, the first order approach is valid.

Using the first order condition approach, the recursive planner's problem is

$$V^u(U) = \max_{b,a,s,U^u,U^e} -b + \beta[p(a)V^e(U^e) + (1-p(a))V^u(U^u)], \tag{21}$$

$s.t.$

$$u(b + s\varpi) - v(s+a) + \beta[p(a)U^e + (1-p(a))U^u] - U = 0, \tag{22}$$

$$\beta p'(a)[U^e - U^u] - v'(s+a) = 0, \tag{23}$$

$$-u'(b + s\varpi)\varpi + v'(s+a) \geq 0, \tag{24}$$

$$(U^e, U^u) \in \mathbf{P}^2, \tag{25}$$

$$s \geq 0, \tag{26}$$

where

$$V^e(U) = \max_{\tilde{U}^u,\tilde{U}^e,w} \quad \omega - w + \beta[\delta V^u(\tilde{U}^u) + (1-\delta)V^e(\tilde{U}^e)]$$

$s.t.$

$$u(w) - v(o) + \beta[(1-\delta)\tilde{U}^e + \delta\tilde{U}^u] - U = 0,$$

$$(\tilde{U}^u, \tilde{U}^e) \in \mathbf{P}^2.$$

This problem is used hereafter to characterize the optimal unemployment insurance with a hidden labor market.[9]

[9] Notice that there is no moral hazard during employment. This implies that promised utility remains constant during employment; i.e., $\tilde{U}^e = U$. To see this, notice that the first order condition with respect to $\tilde{U}^e$ together with the envelope condition yield

$$V'^e(\tilde{U}^e) = V'^e(U),$$

which implies $\tilde{U}^e = U$ provided concavity of $V^e$. The functions $V^u$ and $V^e$ turn out to be strictly concave in all the numerical exercises.

## 2.4 Optimal UI with observable effort

As a benchmark, it is useful to characterize the optimal contract when search effort and participation in the hidden labor market are both observable. The problem is the same as in (21) but now there is no need to introduce the lower bound in promised utilities and constraints (23) and (24).

Let $\lambda$ be the multiplier associated with constraint (22). Then, the first order conditions with respect to $b$ and $U^u$ are

$$(b:) \quad \frac{1}{u'(c)} = \lambda, \tag{27}$$

$$(U^u:) \quad \lambda + V'^u(U^u) = 0, \tag{28}$$

The envelope condition implies

$$V'^u(U) = -\lambda \Longrightarrow V'^u(U^u) = -\lambda'. \tag{29}$$

Thus, combining (46),(47) and (29),

$$\frac{1}{u'(c)} - \frac{1}{u'(c')} = 0 \Longrightarrow c = c'. \tag{30}$$

That is, the first-best allocation implies a constant stream of consumption during unemployment.[10] The intuition is clear that with perfect information regarding individuals actions, there is no need to introduce incentive through consumption dispersion. Thus, the prescribed actions are implemented under full insurance.

Importantly, the first-best allocation is not incentive compatible under hidden effort. This is clear looking at the first order condition with respect to search effort,

$$\underbrace{\beta p'(a)[V^e(U^e) - V^u(U^u)]\lambda^{-1}}_{\text{marginal "external" benefit of } a} + \underbrace{\beta p'(a)(U^e - U^u)}_{\text{mg. "internal" benefit of } a} = \underbrace{v'(a+s)}_{\text{mg. cost of } a}. \tag{31}$$

[10] Using a similar argument, it can be shown that there is also constant consumption across labor status.

The first term of this expression basically states that the planner takes into account how search effort increases the probability of obtaining resources from the individual. With unobservable effort, the individual only considers the last two terms; i.e., equates the marginal internal benefit of search effort with its marginal cost.

Somehow surprisingly, if the planner has control over search effort but not over participation, observability regarding participation in the hidden labor market becomes immaterial. The reason is that once search effort is guaranteed at the social optimum level, the planner and individual's positions regarding the optimal level of participation in the hidden labor market are aligned. This property is formally stated in lemma 3.

**Lemma 3** *If search effort is observable, the first-best allocation is achievable even if the planner cannot observe participation in the hidden labor market.*

Notice that Lemma 3 does not imply that the constraint (24) can be discarded when solving the problem with asymmetric information. When search effort is not observable, the cost and the benefit of an additional unit of participation in the hidden labor market are valued differently by the planner and by the individual. In particular, the planner internalizes the marginal effect of a unit of hidden labor market participation in the cost of providing incentives to make compatible the desired search effort. Because this result emerges from the non-separability between search effort and hidden labor market participation, with a linear effort-cost function, the constraint (24) can be discarded when solving the problem with asymmetric information.

**Lemma 4** *If $v$ is linear, the problem (21) can be solved without considering (24).*

## 2.5 Optimal UI with unobservable effort

Suppose the planner does not observe either search effort or hidden labor market participation. Let $\mu_1$ be the multiplier associated with (23), $\mu_2$ the multiplier associated with (24) and $\phi$ the one associated with (26). As before, let $\lambda$ be the multiplier associated with (22). Finally, let $\xi^u$ and $\xi^e$ be the multipliers associated with constraints (25) for $U^u$ and $U^e$, respectively.

The first order conditions for the planner's problem are

$$\begin{aligned}
&(s:)\ \ \lambda[u'(b+s\varpi)\varpi - v'(s+a)] - \mu_1 v''(s+a) = && (32)\\
&\qquad \mu_2[u''(b+s\varpi)\varpi^2 - v''(s+a)] + \phi, \\
&(b:)\ \ \lambda = \frac{1}{u'(c)} + \mu_2 \frac{u''(c)\varpi}{u'(c)} = \frac{1}{u'(c)} - \mu_2 \rho_a(c)\varpi, && (33)\\
&(U^u:)\ \ \lambda + V'^u(U^u) + \frac{\xi^u}{\beta(1-p(a))} = \mu_1 \frac{p'(a)}{1-p(a)}, && (34)\\
&(U^e:)\ \ \lambda + V'^e(U^e) + \frac{\xi^e}{\beta p(a)} = -\mu_1 \frac{p'(a)}{p(a)}, && (35)\\
&(a:)\ \ p'(a)[V^e(U^e) - V^u(U^u)] + \\
&\qquad \mu_1[\beta p''(a)(U^e - U^u) - v''(s+a)] + \mu_2 v''(s+a) = 0, && (36)
\end{aligned}$$

where $\rho_a$ represents the coefficient of absolute risk aversion. These conditions together with complementary slackness and the corresponding constraints completely characterize the contract solving (21). Using the envelope condition and equations (33)-(34), the following expression can be obtained:

$$\underbrace{\frac{1}{u'(c')} = \frac{1}{u'(c)} - \mu_1 \frac{p'(a)}{1-p(a)}}_{\text{standard in UI models}} + \underbrace{\overbrace{\varpi[\mu_2' \rho_a(c') - \mu_2 \rho_a(c)]}^{\text{from effect of } s \text{ on cost of } a} + \overbrace{\frac{\xi^u}{\beta(1-p(a))}}^{\text{from bound on } U}}_{\text{new terms}}. \tag{37}$$

This equation governs the profile of consumption during unemployment. The hidden labor market adds two terms to this equation. The first term arises because of the effect of participation in the hidden labor market on the cost of search effort.[11] Thus, this term disappears in the case of linear effort-cost function (the multipliers $\mu_2$ and $\mu_2'$ are zero). Similarly, this term is irrelevant as far as the constraint for participation in the hidden labor market is not binding. As will be seen, it will happen during the first months of unemployment. The second term arises because of the bound on the lifetime utility that the system can deliver. Thus, it will be important only for long-term unemployed individuals.

# 3 Qualitative analysis

In this section the cost of effort is defined by

$$v(x) = \alpha x. \tag{38}$$

With this additional assumption, the contract can be characterized analytically.[12]

When the cost of effort is defined as in (38), the dynamic of consumption is characterized by a decreasing path, until a period when it becomes flat. This result is formalized in a set of lemmas.

**Lemma 5** *The following results hold during unemployment:*

1. *When the constraint for promised utility if unemployed (25) does not bind—i.e.,* $U^u > \underline{U}$*—the consumption path is strictly decreasing.*

[11]Notice that the term $\varrho$ in (37) is a function of the absolute coefficient of risk aversion. A similar result is found in an environment with moral hazard and hidden savings. See Abraham and Pavoni (2005).

[12]Some results also hold under a more general specification of $v$. In particular, consider $v(a, s) = g(a)+q(s)$ with $g'(a) > 0$ and $q'(s) > 0$. In this case, (i) the sequence of consumption during unemployment is still decreasing and bounded below, (ii) the lower bound for promised utility is eventually reached, and (iii) when the lower bound is reached, the contract prescribes zero payments and the lower bound for consumption. However, in this case, participation in the hidden labor market does not occur only when the lower bound for promised utility is reached. Notice that Lemma 4 also holds under this specification of $v$.

2. *The sequence of consumption is bounded below by*

$$\underline{c} \equiv u'^{-1}(\alpha/\varpi).$$

*This consumption level is achieved whenever unemployed individuals participate in the hidden labor market.*

3. *For a sufficiently large unemployment spell, the lower bound for promised utility is reached; i.e., there exist a period $t^*$ such $U^u_{t^*} = \underline{U}$.*

If $v$ is linear and $U^u > \underline{U}$, unemployment payments, consumption and promised utility decrease during the unemployment spell. Thus, the system provides dynamic incentives as in Hopenhayn and Nicolini (1997). Consumption is bounded from below by $\underline{c}$ because the agent will obtain this level by participating in the hidden labor market if less than that is prescribed. In fact, linearity in the effort cost function implies that the marginal cost of participation is constant. Thus, every time that the individual participates in the hidden labor market, the marginal benefit must be constant, too. Therefore, consumption must be $\underline{c}$ when participation is positive. Finally, it must be the case that the constraint associated with the lower bound for promised utility eventually binds; otherwise, the contract would prescribe "immiserization" and the lower bound for promised utility would be violated.

Under concavity of the value function, consumption and promised utility can be characterized at the lower bound for promised utility.[13]

**Lemma 6** *If $V^u$ is concave:*

[13] Stating these results conditional on the concavity of the value function and verifying that it is in fact concave in the quantitative results is the same strategy followed by Hopenhayn and Nicolini (1997). As they stated, the restrictions in the contractual problem are non-linear and in general define a set that is non-convex. As a result of the non-convexity of the choice set, it is hard to provide conditions under which the value function solving the contractual problem is concave.

1. *The lower bound for promised utility is a fixed point of the function* $U^u$*; i.e.,* $U^u(\underline{U}) = \underline{U}$*.*

2. *At the lower bound for promised utility, the optimal contract prescribes the consumption's lower bound; i.e.,* $c(\underline{U}) \equiv \overline{c} = \underline{c}$*.*

Once the lower bound for promised utility is reached, the individual can no longer be punished through further reduction in continuation values. Henceforth, the continuation value remains at its lower bound. The cheapest way to deliver $\underline{U}$ is by a constant stream of consumption equal to $\underline{c}$. Together, these results imply that initially, if $U > \underline{U}$, consumption decreases over the unemployment spell until the lower bound for promised utility is reached. From that period on, consumption remains constant at its lower bound.

Importantly, with a hidden labor market, consumption does not necessarily equal unemployment payments. Hence, the characterization of consumption is not enough to identify the optimal path for unemployment payments. The dynamics of unemployment payments are characterized in the following proposition.

**Proposition 1** *Suppose* $V^u$ *is concave. Then, unemployment insurance payments decrease until the lower bound for promised utility is reached; after that moment they become zero; i.e,* $b_{t+1} < b_t$ *when the constraint (25) does not bind; if constraint (25) binds,* $b_{t+1} = 0$*.*

To see that payments initially decrease, remember that in the equation governing the profile of consumption during unemployment, (37), the first extra term disappears in the case of linear effort-cost function. Additionally, the other extra term also vanishes because the bound on promised utility is not binding. Then, this initial phase resembles Hopenhayn and Nicolini (1997), so unemployment payments are decreasing. Once the lower bound for promised utility is reached, the payments are zero because this is the cheapest way to provide that level of lifetime utility (remember that taxes on unemployed individuals are not allowed). Notice that Proposition 1 implies that the optimal contract eventually prescribes an abrupt decline in payments. Despite this sudden drop in payments, consumption remains smooth as the worker compensates the decrease in the payments with income from the hidden labor market.

**Corollary 1** *Participation in the hidden labor market takes one of two values:*

1. *When the lower bound for promised utility is not binding,* $s = 0$*.*
2. *When the lower bound for promised utility is binding,* $s = s^* > 0$*.*

The intuition behind corollary 1 is clear. If promised utility is above its lower bound, consumption is higher than $\underline{c}$ and hence, there is no incentive for participation in the hidden labor market—the marginal costs are higher than the marginal benefits of participation. Once promised utility reaches its lower bound, payments are zero so the level of participation in the hidden labor market is the one setting the marginal costs equal to marginal benefits, $s^* > 0$.

Previous corollary highlights a key property of the optimal contract in the case of linear effort-cost function; namely, *initially, the planner does not encourage participation in the hidden labor market*. The explanation for this somehow unexpected result is that, once the planner induces participation, it completely loses the ability to punish the individual through reduction in consumption. Moreover, as it will be clear in the numerical exercise, this property is still present, to some extent, in the case of non-linear effort-cost function.

In summary, there are two different phases during unemployment. During the first phase, payments (consumption) decrease smoothly without reaction of hidden labor market participation. It is because the payments are relatively high and hence the marginal cost of participation is higher than its marginal benefit. However, as payments decline, the marginal benefit of participation increases, and participation is eventually optimal. At that point the last phase of unemployment starts. In this phase, with positive participation, linearity and incentive compatibility uniquely determine the consumption level ($\underline{c}$). At that level of consumption, promised utility can be provided with zero payments (any positive payments deliver a higher lifetime utility). Thus, during this phase, zero payments become optimal.

# 4 Quantitative analysis

The quantitative analysis in this section shed light on the case of non-linear effort-cost function. The first subsection presents the calibration strategy. The next subsection describes the dynamics of unemployment benefits, job search effort and informal sector participation under the optimal contract. Finally, the path of unemployment payments is analyzed further with a comparison with alternative environments.[14]

## 4.1 Calibration

Functional forms are standard. Preferences over consumption goods are described by a CRRA function with relative risk aversion coefficient $\sigma$, the job finding probability function is $p(a) = 1 - \exp(-\rho a)$, and the cost of effort function is $v(a) = a^\gamma/\gamma$.

A key preliminary question regarding the calibration is, Which economy should be considered for the benchmark calibration? Quantitative research usually uses the U.S. economy. However, this environment naturally applies to developing countries, where the unemployment rate and the informal sector size are high. When working with developing countries, certain problems arise, however. First, it is hard to find reliable data for the informal sector.

[14]More details on the calibration and the numerical algorithm are presented in the online Appendix.

More importantly, there is not accurate information about unified and explicitly designed unemployment insurance systems. Given this restriction, a valid alternative is the Spanish economy at the end of the '90s. The advantage of this choice is that it combines the importance of unemployment risk and the informal sector with the availability of information.[15]

Most of the parameters are taken from previous literature. Each period is a month, so the discount factor is $\beta = 0.994$ (annual interest rate of 7.5%). Individuals' productivity, $\omega$, and effort in the formal labor market, $o$, are normalized to 1. Data about average tenure in Spain prescribes $\delta = 0.015$.[16] The relative risk aversion coefficient, $\sigma$, is set equal to 2, which is a common value in the real business cycle literature.[17]

There are two more parameters that need to be calibrated: the parameter in the job-finding probability function, $\rho$, and the wage in the informal sector, $\varpi$. The calibration strategy uses the actual unemployment insurance scheme instead of the optimal contract. For plausible combinations of $\rho$ and $\varpi$, the stationary distribution is computed for an economy in which agents take as given the Spanish unemployment insurance scheme. The unemployment rate and the informal sector size are computed using this distribution. These outcomes are then compared with those in the data. The calibration consists of finding values of $\rho$ and $\varpi$ compatible with the targets. Using the actual Spanish unemployment insurance scheme rather than the optimal is natural since the unemployment rate and the informal sector size in the data are outcomes of the individuals' reactions to the actual system.

The calibration matches the unemployment rate and informal sector size in Spain for

[15] A possible concern is that in the last 20 years unemployment rate, informal sector participation, and labor market regulations have changed significantly in Spain. Although this concern may imply that the conclusions of this exercise may not apply to the current Spanish economy, it does not invalidate the application for developing countries, whose current labor market structure is similar to that used in the calibration.

[16] According to Arranz and Garca-Serrano (2004) the average tenure in the private sector is around 6 years.

[17] Different values have been used in the literature on optimal unemployment insurance. Pavoni (2007) works with log preferences ($\sigma = 1$) while Hopenhayn and Nicolini (1997) set $\sigma = 0.5$.

1997. Unemployment rate, $\mu$, was 18.5 percent while production in the informal sector as a proportion of the GDP, $\zeta$, was 23 percent—Schneider and Enste (2000). Table 1 summarizes our calibration. With these parameters the artificial economy reproduces $\mu = 0.1787$ and $\zeta = 0.2326$, very close to the targets.

Table 1: Calibration strategy

| Parameter | Value | Description | Basis and Targets |
|---|---|---|---|
| $\beta$ | 0.994 | Discount factor | Interest rate |
| $\sigma$ | 2 | Relative risk aversion coefficient | Standard |
| $\gamma$ | 2 | Search cost elasticity | Standard |
| $\omega$ | 1 | Productivity in formal sector | Normalization |
| $o$ | 1 | Effort when employed | Normalization |
| $\delta$ | 0.015 | Separation rate | Average tenure |
| $\rho$ | 0.045 | Parameter in probability of finding a job | Unemployment rate |
| $\varpi$ | 0.415 | Wage in informal sector | Informal sector size |

## 4.2 Optimal policy

The inclusion of a hidden labor market is crucial in determining the optimal UI scheme. Figure 1 shows the path for search effort and hidden labor market participation and Figure 2 depicts the optimal unemployment payments and consumption. In both figures, the initial promised utility is associated with a "typical" unemployed individual with whom the planner breaks even. Both figures are separated into three phases that are important for understanding the effect of the hidden labor market. The analysis of those phases is postponed until the next subsection.

Figure 1 indicates that the optimal participation in the hidden labor market is initially zero and search effort increases with the unemployment spell. This phase resembles the results in Hopenhayn and Nicolini (1997). After approximately 2.5 years of unemployment, when effort peaks, participation jumps and search effort drops. After that point, participa-

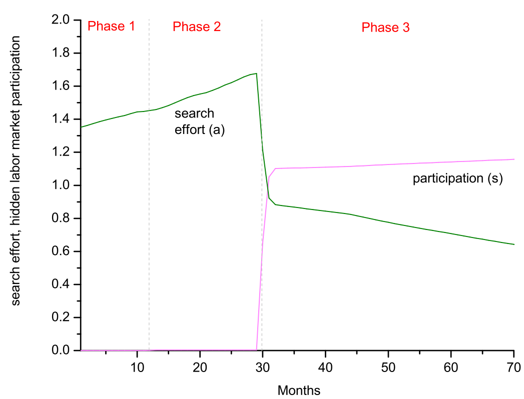


Figure 1: The dynamics of search and participation.

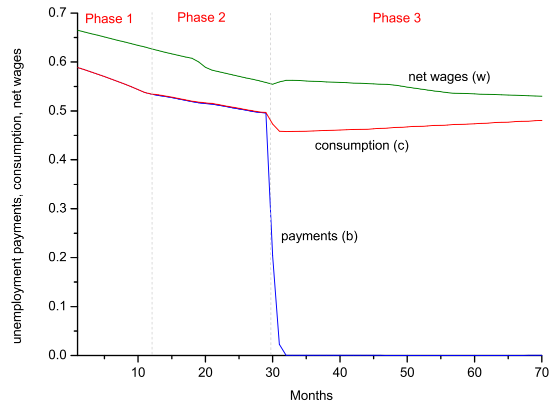


Figure 2: The dynamics of payments, consumption, and net wages.

tion increases and search effort decreases gradually toward fairly steady levels. Some aspects are remarkable from this behavior. First, required effort is not monotonically increasing in the unemployment spell. The reason is that in order to induce an increase in participation in the hidden labor market, it is optimal to reduce the search effort such that the marginal cost of inducing participation falls. Second, in the early phase of unemployment, individuals are fundamentally searchers with no participation in the hidden labor market. Thus, it is very likely that unemployed individuals find a job before they start to participate in the hidden labor market.

Figure 2 shows that unemployment payments and consumption are equal and decrease smoothly while the agent is not participating in the hidden labor market. Net wages are also decreasing during the unemployment spell. This means that taxes will be higher for worker that spent more time unemployed.[18] When participation jumps, payments drop to zero and consumption is slightly increasing thereafter. The rise in consumption is related to

[18]The same result was first found by Hopenhayn and Nicolini (1997).

the decrease in promised utility during this phase. Since payments are at zero, decreasing promised utility means that taxes after reemployment will be higher. As a consequence, unemployed individuals substitute search effort with hidden labor market participation and, therefore, enjoy higher consumption. The next subsection provides further analysis of the optimal path of unemployment payments.

### 4.3 More on the optimal profile of unemployment payments

In order to identify the key elements shaping the payment profile in each phase, it is useful to compare the optimal paths of payments under alternative environments. The first environment is the same presented in Pavoni (2007), parameterized according to Table 1. The planner's problem in this case is a simplification of problem (21). In particular, the constraint (24) is excluded and participation in the hidden labor market is set at zero, $s = 0$. The second environment considered is one in which participation in the hidden labor market is observable by the planner. The problem is basically (21) but without constraint (24). This second alternative environment differs from Pavoni (2007) in that $s$ is not set at zero but chosen optimally. In the three environments, the lower bound for promised utility is given by the system (7)-(8).

Figure 3 shows the unemployment payments profile in each case. In addition, Figure 4 shows the left-hand side of constraint (24) with a negative sign. A positive value in this figure indicates that individuals want to participate in the hidden labor market more than that which is prescribed by the contract.[19] This variable, together with the level of participation in the hidden labor market, define three phases during the unemployment spell.[20] What does

[19]This term is meaningless for Pavoni's case and is therefore excluded.

[20]Let this variable be refereed as $-LHS_{24}$. Then, in *Phase 1*, $-LHS_{24} < 0$ and $s = 0$; in *Phase 2*, $-LHS_{24} = s = 0$; and finally, in *Phase 3*,$-LHS_{24} = 0$ and $s > 0$.

the payments scheme look like in each environment? First, let us focus on Pavoni (2007). Unemployment payments decrease during the unemployment spell, and, more importantly, as promised utility approaches its lower bound, payments decrease faster and faster. Eventually, the individual cannot be further punished and the payments stabilize at a positive value. For the observable-$s$ environment, the payment profile becomes steeper with the unemployment spell. The main difference relative to Pavoni (2007) is that now payments drop to zero. This can happen because in this context individuals can smooth out consumption by participating in the hidden labor market. In terms of incentive compatibility, it is remarkable that the plan prescribed by the optimal contract in this environment is not implementable if $s$ is not observable. As Figure 4 shows, the constraint (24) is violated by this contract. Now focus on the benchmark case. In phase 1, payments decrease even faster than in the alternative environments. This happens while the constraint (24) is not binding with zero hidden labor market participation. However, eventually the constraint (24) becomes binding and a fast decrease in payment is not compatible with zero participation. This is where the contract with a hidden labor market differs from the other environments. In this case, the payment profile becomes flatter to prevent the individual from participation in the hidden labor market. The flattening in the payment scheme is the main implication of the non-observability of hidden labor market participation. During all of phase 2, participation is set at zero but the constraint (24) is binding. Here, incentives are mainly introduced by decreasing promised utility. Eventually, the accumulated drop in promised utility is only implementable by an abrupt decline, down to zero, in unemployment payments. The elimination of payments and the associated jump in hidden labor market participation (see Figure 1) represent the beginning of phase 3. As in the case in which participation in the hidden labor market is observable, the optimal contract provides no payment for individuals with sufficiently large unemployment spells. In sum, the non-observability of the hidden labor market is responsible for the flattening in the payment schedule (phase 2), while the existence of a hidden labor market (that can be used as a source of income) is responsible for the optimality of the elimination in unemployment payments for a sufficiently long unemployment spell.

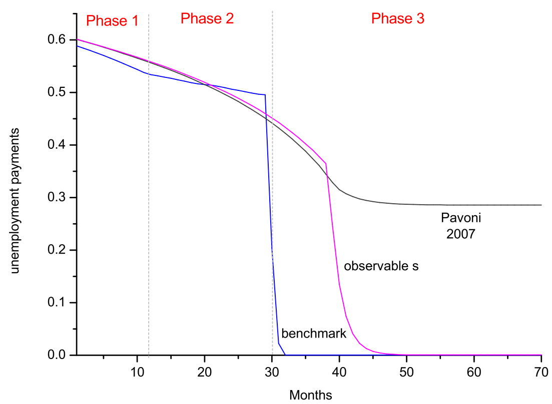


Figure 3: Payment profile.

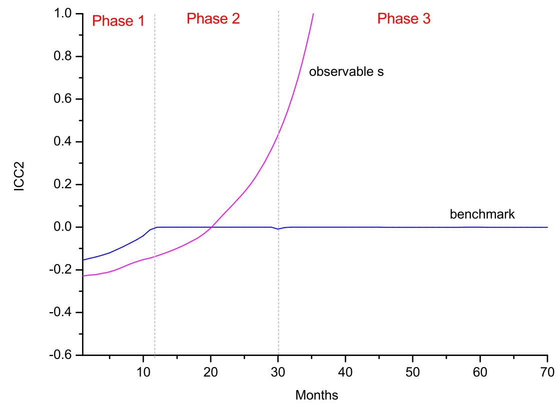


Figure 4: Constraint (24).

Phase 2 and 3 are the novel features in the payment profile relative to the standard optimal unemployment insurance problem. The question is, how relevant are these two phases from a quantitative point of view? In order to answer this question, the distribution of workers across the different phases that arise under the optimal contract with a hidden labor market are considered.[21] Table 2 reports the results for some selected groups of potential experience.[22]

Two aspects are remarkable from Table 2. First, phase 2 is quantitatively the most important. Indeed, between 60 percent and 70 percent of unemployed individuals with five

[21] To compute this distribution, start the economy with a large number of unemployed individuals to whom the contracts deliver the same lifetime utility. Set this initial lifetime utility such that the planner breaks even with each individual. Then, simulate large paths for labor status, as well as for continuation values. If the individual is unemployed, also simulate paths for participation in the hidden labor market and effort.

[22] Potential experience is equivalent to years in the labor market.

Table 2: Conditional distribution of unemployed individuals across UI phases

| Potential experience | Phase 1 | Phase 2 | Phase 3 |
|---|---|---|---|
| 5 | 0.260 | 0.602 | 0.138 |
| 10 | 0.235 | 0.657 | 0.108 |
| 15 | 0.230 | 0.661 | 0.109 |
| 20 | 0.225 | 0.671 | 0.103 |
| 25 | 0.218 | 0.677 | 0.105 |
| 30 | 0.211 | 0.690 | 0.099 |
| 35 | 0.207 | 0.695 | 0.098 |

or more years in the labor market are in this phase. Second, there exists a significant number of individuals receiving no payments. In fact, around 10 percent of unemployed individuals with five or more years in the labor market are receiving zero payments.

Finally, it is worth noticing that the preceding analysis suggests that a simple 2-step system looks closer at the optimal payment scheme than previous literature suggested. The initial two phases have to be generous enough to keep the individual out of the secondary labor market, while the last step is characterized by no payments at all. This is important because many countries have unemployment protection programs with that structure (e.g. United States).

# 5 Conclusions

The problem of the optimal unemployment insurance has been studied from the perspective of a principal-agent framework since the pioneering work of Shavell and Weiss (1979). Early literature mainly focuses on the moral hazard problem arising under non-observability of search effort. More recent works also emphasize the non-observability of assets. However, the implications of the existence of a hidden labor market were unknown. This was an important omission in the unemployment insurance literature because the incorporation of a hidden labor market has non-trivial consequences. Moreover, understanding the role of a hidden labor market is highly relevant because it represents a sizeable segment of the labor market in many countries. This paper fills this gap by providing an environment in which unemployed individuals can simultaneously search for a job and participate in the hidden labor market. There are three main findings. First, the presence of a hidden labor market imposes an endogenous lower bound for promised utility and, unless the effort-cost function is linear, an extra incentive-compatibility constraint must be added. Second, with the linear effort-cost function, there are two important phases. In the initial phase, consumption decreases during unemployment and individuals do not participate in the hidden labor market. In the next phase, which starts after sufficiently large unemployment spells, consumption remains constant, payments become zero and participation jumps to a positive constant. Third, with a non-linear effort-cost function, there exist an intermediate phase. There, participation in the hidden labor market is zero but individuals are indifferent between increasing participation or not (the incentive compatibility constraint for $s$ is binding). This implies that the payment profile must be relatively flatter to avoid participation and to keep search effort high. This phase, together with the subsequent drop in payments, differentiate this work from previous literature. As a whole, the optimal unemployment insurance with a hidden labor market is simple—with a relatively flat phase and with no payments for long-term unemployed individuals—and looks similar to unemployment protection programs in many countries.

## References

Abraham, A., and N. Pavoni (2005): "Efficient Allocations with Moral Hazard and Hidden Borrowing and Lending," University of Rochester.

Abreu, D., D. Pearce, and E. Stacchetti (1990): "Toward a Theory of Discounted Repeated Games with Imperfect Monitoring," *Econometrica*, 58(5), 1041–1064.

Arranz, J. M., and C. Garca-Serrano (2004): "Qué ha Sucedido con la Estabilidad del Empleo en España? Un Análisis Desagregado con Datos de la EPA:1987-2003," Papeles de Trabajo, 4/04.

Banerjee, B. (1983): "The Role of the Informal Sector in the Migration Process: A test of Probabilistic Models and Labour Market Segmentation for India," *Oxford Economic Papers*, 35, 399–422.

Fortin, B., N. Marceau, and L. Savard (1997): "Taxation, Wage Controls and the Informal Sector," *Journal of Public Economics*, 66, 239–312.

Green, E. J. (1987): *Lending and the Smoothing of Uninsurable Income. In Contractual arrangements for intertemporal trade. Edited by Edward C.Prescott and Neil Wallace.* University of Minnesota Press, minnesota studies in macroeconomicsseries, vol. 1. edn.

Hansen, G. D., and A. Imrohoroglu (1992): "The Role of Unemployment Insurance in an Economy with Liquidity Constraints and Moral Hazard," *Journal of Political Economy*, 100(1), 118–42.

Hopenhayn, H., and J. Nicolini (2005): "Optimal Unemployment Insurance and Employment History," Manuscript.

Hopenhayn, H. A., and J.-P. Nicolini (1997): "Optimal Unemployment Insurance," *Journal of Political Economy*, 105(2), 412–438.

ILO (2004): *Global Employment Trends*. International Labour Office.

Johnson, S., D. Kaufmann, and P. Zoido-Lobaton (1998): "Regulatory Discretion and the Unofficial Economy," *American Economic Review*, 88, 387–392.

Kocherlakota, N. (2004): "Figuring out the Impact of Hidden Savings on Optimal Unemployment Insurance," *Review of Economic Dynamics*, 7(3), 541–554.

Pavoni, N. (2007): "On optimal unemployment compensation," *Journal of Monetary Economics*, 54(6), 1612–1630.

Phelan, C. (1995): "Repeated Moral Hazard and One-Sided Commitment," *Journal of Economics Theory*, 2(66), 488–506.

Schneider, F., and D. Enste (2000): "Shadow Economies: Size, Causes and Consequences," *Journal of Economic Literature*, XXXVIII, 77–114.

Shavell, S., and L. Weiss (1979): "The Optimal Payment of Unemployment Insurance Benefits over Time," *Journal of Political Economy*, 87(6), 1347–1362.

Spear, S., and S. Srivastava (1987): "On Repeated Moral Hazard with Discounting," *Review of Economic Studies*, 53, 599–617.

Thomas, J., and T. Worral (1990): "Income Fluctuations and Asymmetric Information: An Example of Repeated Principal-Agent Problem," *Journal of Economic Theory*, 51(2), 367–390.

VODOPIVEC, M., AND D. RAJU (2002): “Income Support Systems for the Unemployed: Issues and Options,” Social Protection Discussion Paper Series, No. 0214.

WANG, C. (1995): “Dynamic Insurance with Private Information and Balanced Budgets,” *Review of Economics Studies*, 4(62), 577–596.

WANG, C., AND S. WILLIAMSON (1996): “Unemployment Insurance with Moral Hazard in a Dynamic Economy,” *Carnegie-Rochester Conference Series on Public Policy*, (44), 1–41.

# Appendix

## All proofs

### Proof of lemma 1

**Proof:** First notice that that optimality requires

$$m_{t+1} = m_t = e \Rightarrow \mathcal{U}_t(\mathcal{W}; h^t) = \mathcal{U}_{t+1}(\mathcal{W}; h^{t+1}). \tag{39}$$

Now, suppose by contradiction that at any arbitrary period of time T, the constrained efficient contract $\mathcal{W}^* = (\mathcal{P}^*, \mathcal{I}^*)$ delivers a promised utility $\mathcal{U}_T(\mathcal{W}^*; h^T) < \underline{U}$. We prove this violates incentive compatibility by constructing a feasible deviation for any possible labor status $m_T$ providing a promised utility higher than $\underline{U}$.

- If $m_T = u$, consider the following feasible deviation $\widetilde{\mathcal{P}}$, with $(\widetilde{a}_t(h^t), \widetilde{s}_t(h^t)) = (0, s^*)$ if $m_t = u$, and $(\widetilde{a}_t(h^t), \widetilde{s}_t(h^t)) = (0, 0)$ if $m_t = e$. Clearly, $\widetilde{\mathcal{P}} \in \mathbf{A}$ and $\mathcal{U}_T((\widetilde{\mathcal{P}}, \mathcal{I}); h^T) = \underline{U} > \mathcal{U}_T((\mathcal{P}, \mathcal{I}); h^T)$. Hence, the contract $\mathcal{W}^*$ is not incentive compatible and, therefore, cannot be the constrained efficient contract.

- If $m_T = e$, define $\widehat{T}$ as the first period of that employment spell. By property (39), $\mathcal{U}_{\widehat{T}}(\mathcal{W}^*; h^{\widehat{T}}) = \mathcal{U}_T(\mathcal{W}^*; h^T)$. Now consider the situation in $\widehat{T}-1$. By definition, $m_{\widehat{T}-1} = u$. Moreover, since $m_{\widehat{T}} = e$, it must be the case that the contract prescribes $a_{\widehat{T}-1}(\cdot) > 0$. However, if $\mathcal{U}_T(\mathcal{W}^*; h^T) < \underline{U}$, it must be the case that $a_{\widehat{T}-1}(\cdot) > 0$ is not compatible incentive because a deviation with $\widetilde{a}_{\widehat{T}-1}(\cdot) = 0$ implies higher promised utility than $\underline{U}$.

To see this, notice that since $v$ is strictly increasing,

$$\mathcal{U}_{\widehat{T}-1}(\mathcal{W}^*; h^{\widehat{T}-1}) = u(c_{\widehat{T}-1}) - v(s_{\widehat{T}-1} + a_{\widehat{T}-1}) +$$
$$\beta[p(a_{\widehat{T}-1})\mathcal{U}_{\widehat{T}}(\mathcal{W}^*; h^{\widehat{T}}|m_{\widehat{T}} = e) + (1 - p(a_{\widehat{T}-1}))\mathcal{U}_{\widehat{T}}(\mathcal{W}^*; h^{\widehat{T}}|m_{\widehat{T}} = u)]$$
$$< u(c_{\widehat{T}-1}) - v(s_{\widehat{T}-1}) +$$
$$\beta[p(a_{\widehat{T}-1})\mathcal{U}_{\widehat{T}}(\mathcal{W}^*; h^{\widehat{T}}|m_{\widehat{T}} = e) + (1 - p(a_{\widehat{T}-1}))\mathcal{U}_{\widehat{T}}(\mathcal{W}^*; h^{\widehat{T}}|m_{\widehat{T}} = u).]$$

We showed in the first part of this proof that during unemployment, promised utility must be at least $\underline{U}$; i.e.,

$$\underline{U} \leq \mathcal{U}_{\widehat{T}-1}(\mathcal{W}^*; h^{\widehat{T}}|m_{\widehat{T}} = u).$$

Then if $\mathcal{U}_{\widehat{T}-1}(\mathcal{W}^*; h^{\widehat{T}}|m_{\widehat{T}} = e) < \underline{U}$,

$$\mathcal{U}_{\widehat{T}-1}(\mathcal{W}^*; h^{\widehat{T}-1}) < u(c_{\widehat{T}-1}) - v(s_{\widehat{T}-1}) + \beta\mathcal{U}_{\widehat{T}}(\mathcal{W}^*; h^{\widehat{T}}|m_{\widehat{T}} = u). \tag{40}$$

The last expression indicates that $\mathcal{W}^*$, which prescribes $a_{\widehat{T}-1} > 0$, is not incentive compatible because a deviation with $\widetilde{a}_{\widehat{T}-1} = 0$ provides a higher utility to the individual. Hence, $\mathcal{W}^*$ cannot be the constrained efficient contract.

■

## Proof of lemma 2

**Proof:** Take $U \geq \underline{U}$ and consider the contract $\widetilde{\mathcal{W}}$. For each $t$ and $h^t$ it assigns $b_t(h^t) = \tilde{b}$, $s_t(h^t) = \tilde{s}$, $a_t(h^t) = 0$, and $w_t(h^t) = \tilde{w}$. Define $(\tilde{b}, \tilde{w}, \tilde{s})$ by

$$u'(\tilde{b} + \tilde{s}\varpi)\varpi = v'(\tilde{s}), \tag{41}$$

$$U = \frac{u(\tilde{b} + \tilde{s}\varpi) - v(\tilde{s})}{1-\beta}, \text{ and} \tag{42}$$

$$u(\tilde{b}+\tilde{s}\varpi)-v(\tilde{s})=u(\tilde{w})-v(o). \tag{43}$$

By construction, $u(c_t(h^t))-v(x_t(h^t))$ is constant across time and histories and $\mathcal{U}_t(\widetilde{\mathcal{W}};h^t)=U$. It remains to be proven that $\widetilde{\mathcal{W}}\in\mathbf{C}$. Since deviations are only possible during unemployment and a contract is incentive compatible against any possible deviation if and only if it is incentive compatible against a one-period deviation, incentive compatibility implies that for each $t$ and each $h^t$

$$\begin{gathered} U\geq\max_{(s,a)}u(\tilde{b}+s\varpi)-v(s+a)+\beta U,\\ \text{s.t. } a\geq 0, s\geq 0, \text{ and } s+a\leq\vartheta. \end{gathered} \tag{44}$$

In (44) we impose the fact that for each history and period, $\mathcal{U}_t(\widetilde{\mathcal{W}};h^t)=U$. Clearly, (44) is satisfied because the solution of the maximization problem implies $a=0$ and $s=\tilde{s}$. Hence, $\widetilde{\mathcal{W}}\in\mathbf{C}(U;h^0)$. ∎

## Proof of lemma 3

**Proof:** The proof is straight forward. The first best contract is characterized by

$$(s:)\quad -u'(b+s\varpi)\varpi+v'(s+a)\geq 0, \tag{45}$$

$$(b:)\quad \frac{1}{u'(c)}=\lambda, \tag{46}$$

$$(U^u:)\quad \lambda+{V'}^u(U^u)=0, \tag{47}$$

$$(U^e:)\quad \lambda+{V'}^e(U^e)=0, \tag{48}$$

$$(a:)\quad \beta p'(a)[V^e(U^e)-V^u(U^u)]=\lambda[v'(a+s)-\beta p'(a)(U^e-U^u)]. \tag{49}$$

Under observability of effort, (23) is not relevant while (24) is satisfied by (45). ∎

## Proof of lemma 4

**Proof:** Consider the following *Relaxed Problem*:

$$V_R^u(U) = \max_{b,e,s,U^e,U^u} -b + \beta[p(a)V^e(U^e) + (1-p(a))V_R^u(U^u)] \tag{50}$$

$$s.t.$$

$$u(b + s\varpi) - v(s+a) + \beta[p(a)U^e + (1-p(a))U^u] - U = 0 \tag{51}$$

$$\beta p'(a)[U^e - U^u] - v'(s+a) = 0 \tag{52}$$

$$(U^e, U^u) \in \Phi^2 \tag{53}$$

$$s \geq 0.$$

We need to prove that if $v$ is linear, the solution for (50) also solves (21). To that end, denote $D(U)$ the set of all $(b, a, s, U^u, U^e)$ satisfying (23),(24),(22), (25), and non-negativity for $s$. Likewise, denote $D_R(U)$ the set of all $(b, a, s, U^u, U^e)$ satisfying previous restrictions but (24). Clearly $D(U) \subseteq D_R(U)$. Hence, $V_R^u(U) \geq V^u(U)$. Notice however that

$$(\tilde{e}, \tilde{s}, \tilde{U}^e, \tilde{U}^u) \in \text{argmax}_{D_R(U)} V_R^u(U)$$

satisfies (24). To see this, take first order condition with respect to $s$ and notice that $\lambda > 0$ and $\phi \geq 0$. In consequence, $(\tilde{e}, \tilde{s}, \tilde{U}^e, \tilde{U}^u) \in D(U)$ and solves (21); otherwise, it will not be true that $V_R^u(U) \geq V^u(U)$. ■

## Proof of lemma 5

**Proof:**

1. First observe that $\mu_2 = 0$.[23] To see this, notice that if $s = 0$, constraint (24) does not bind and, by complementary slackness, $\mu_2 = 0$. On the other hand, if $s > 0$,

[23] This is consistent with lemma (4), which indicates that with a linear effort-cost function, incentive compatibility constraint (24) is not needed to formulate the constrained planner's problem.

equation (32) implies $\mu_2 u''(b+s\varpi)\varpi = 0$ and then $\mu_2 = 0$. Now, manipulating first order conditions, and letting $\xi^u = 0$, we find

$$\frac{1}{u'(c_t)} - \frac{1}{u'(c_{t+1})} = \mu_1 \frac{p'(a)}{1-p(a)} > 0. \tag{54}$$

Finally, by strict concavity of $u$ we have $c_t > c_{t+1}$.

2. The lower bound may be violated only if $b < \underline{c}$. In that case, $s = 0$ is not incentive compatible. Since participation in the hidden labor market must be optimal from the agent's point of view, it will solve $u'(b+s\varpi)\varpi = \alpha$. This implies that whenever the unemployed individual participates in the hidden labor market, $c = b + s\varpi = \underline{c}$.

3. Suppose by contradiction that constraint (25) never binds. Then, we can write problem (21) without it. Moreover, by corollary (1), $s_t = 0$ for each $t$. As a consequence, problem (21) collapses into the problem studied by Hopenhayn and Nicolini (1997) with no hidden labor market participation and no relevant bound for promised utility. As shown in Proposition 3 in Pavoni (2007), in this environment $U_t$ falls below any bound with positive probability. This contradicts lemma (1).

■

## Proof of lemma 6

**Proof:**

1. Let $U = \underline{U}$, using the first-order condition with respect to $U^u$ and envelope condition, we get

$$V'^u(U^u) - V'^u(\underline{U}) = \mu_1 \frac{p'(a)}{(1-p(a))} - \frac{\xi^u}{\beta(1-p(a))}.$$

Clearly $U^u < \underline{U}$ is not possible because it violates (25). Suppose then that $U^u > \underline{U}$. This implies (25) does not bind and hence $\xi^u = 0$. However if $\xi^u = 0$, the concavity of $V^u$ implies $U^u \leq \underline{U}$, which contradicts our assumption that $U^u > \underline{U}$. Therefore, it must be that $U^u = \underline{U}$.

2. Previous results showed that there exists a $t^*$ such that $U_t^u = \underline{U}$ and $c_t = c(\underline{U}) \equiv \overline{c}$ for each $t > t^*$. By lemma (5), $\overline{c} \geq \underline{c}$, which rules out $\overline{c} < \underline{c}$. We prove $\overline{c} > \underline{c}$ is not a solution. Suppose by contradiction that $\overline{c} > \underline{c}$. Since $u'(\overline{c})\varpi < \alpha$, at this point $s = 0$. Let $a$ be the search effort prescribed in the optimal contract. We can write

$$u(\overline{c}) - v(a) + \beta[p(a)U^e + (1 - p(a))\underline{U}] \geq$$
$$u(\overline{c}) + \beta\underline{U} > u(\underline{c}) - v(s^*) + \beta\underline{U} \equiv \underline{U}.$$

The first inequality arises form the fact that incentive compatibility requires that the utility delivered by the contract is not lower than the one associated with any feasible deviation, in particular with $s = a = 0$. The second *strict* inequality arises form our assumption that $\overline{c} > \underline{c}$ and from the fact that $v$ and $u$ are strictly increasing. However, this implies that in the contract the planner is delivering a lifetime utility strictly higher than the one promised, which is not optimal. Hence $\overline{c} = \underline{c}$.

■

## Proof of proposition 1

**Proof:** When the lower bound for $U$ does not bind, consumption is strictly decreasing and above $\underline{c}$. Since in this phase $s_t = 0$, $b = c$ and, therefore, unemployment payments also decrease. When $U = \underline{U}$, $b$ is constant at $\underline{b}$ because $U^u(\underline{U}) = \underline{U}$. Suppose, by contradiction,

$\underline{b} > 0$. By incentive compatibility, the contract must deliver a utility consistent with an individual's best response, that is, the lifetime utility is

$$\tilde{U} \equiv \max_{a,s} u(b + \varpi s) - v(a + s) + \beta[p(a)U^e + (1 - p(a))\underline{U}].$$

Moreover, this utility must be as good as any deviation, in particular one in which search intensity is set to zero and $s$ is set optimally. That is,

$$\tilde{U} \geq \max_s u(b + \varpi s) - v(s) + \beta\underline{U} \equiv \hat{U}.$$

Since $u$ is strictly increasing,

$$\hat{U} > \max_s u(\varpi s) - v(s) + \beta\underline{U} \equiv \underline{U}.$$

This implies that there exists a feasible reduction in $b$ that increases the planner's surplus. Therefore $b > 0$ is not optimum. ■

## Proof of corollary 1

**Proof:**

1. Suppose by contradiction that at a certain period $s_t \neq 0$ but the lower bound for promised utility has not been reached. By lemma (2), $c_t = \underline{c}$, but since $U^u > \underline{U}$, by lemma (1), $c_{t+1} < c_t = \underline{c}$ which is a contradiction.

2. The proof is trivial. When $U = \underline{U}$, b=0. Then, by incentive compatibility for $s$, $s = s^*$.

■

# Numerical algorithm

1. Define a grid for promised utility: $\mathbf{U} = \{U_1, ..., U_N\}$. The range of $U$ is set such that $U_1 = \underline{U}$ and $U_N$ is big enough to guarantee that the value functions become negative.

2. Define search effort grids for effort and participation: $\mathbf{E_f} = \{a_1, ..., a_M\}$ and $\mathbf{S} = \{s_1, ..., s_M\}$, where $a_1 = s_1 = 0$, and $a_M$ and $s_M$ are big enough to avoid grid-dependent corner solutions.

3. Make an initial guess $V^{u,0}(U) = V^{u,FB}(U)$ for each $U \in \mathbf{U}$, where $V^{u,FB}(U)$ is the first best planner's surplus. For values out of the grid, use a piece-wise linear interpolation.

4. Iteration $i$ starts. Get $V^{u,i+1} = TV^{u,i}$ as follow:

   (a) Suppose $s = 0$. Use equation (33) to obtain $b$. Given $b$, for each $a \in \mathbf{E_f}$ use (23) and (22) to solve for the values of $U^e$ and $U^u$. Choose the combination $\{a, b, U^e, U^u\}$ maximizing (21). Check if (24) is satisfied for $s = 0$. If it is satisfied, the maximization ends. If not, go to the next step.

   (b) Look for the best solution $\{a, s\} \in \mathbf{E_f} \times \mathbf{S}$, using (23), (24) and (22) to solve for the corresponding $b$, $U^e$ and $U^u$.

5. Check convergency. That is, given a metric $d$, evaluate $d(V^{u,i}, V^{u,i+1}) > \epsilon$. If so, let $V^{u,i} = V^{u,i+1}$ and go to (4); otherwise end.

# More on the calibration strategy

In Spain, unemployed individuals qualify for benefits if they have contributed to social security over a minimum period in the previous years. Depending on their employment duration, unemployed individuals receive payments from 4 months to a maximum of 2 years. After that period, they qualify for an unemployment subsidy. A typical individual receives a subsidy equivalent to 70 percent of the previous wage (70 percent replacement ratio) during the first 6 months, and a 60 percent replacement ratio for the next 18 months. Unemployment subsidy is 80 percent of the minimum wage. This implies a replacement ratio of 30 percent after two years of unemployment.[24] Taxes financing the system are set at 5 percent.

Individuals decide effort and hidden labor market participation to maximize their lifetime utility, taking as given the actual unemployment insurance scheme. This scheme is represented by $(\tau, \{b_n\}_{n=1}^{\infty})$, where subindex $n$ refers to unemployment duration. Since $b_n$ is constant for $n \geq n^* = 25$, the agent's problem has the following recursive representation:

$$\hat{V}_n^u = \max_{a,s} u(b_n + s\varpi) - v(a+s) + \beta[p(a)\hat{V}^e + (1-p(a))\hat{V}_{n+1}^u], \forall n \in \{0, 1, ..., n^* - 1\}$$

$$\hat{V}_{n^*}^u = \max_{a,s} u(b_n + s\varpi) - v(a+s) + \beta[p(a)\hat{V}^e + (1-p(a))\hat{V}_{n^*}^u], \forall n \geq n^*$$

$$\hat{V}^e = u(\omega(1-\tau)) - v(o) + \beta[(1-\delta)\hat{V}^e + \delta\hat{V}_0^u],$$

where $\hat{V}_n^u$ is the lifetime utility of an unemployed agent with unemployment spell $n$, and $\hat{V}^e$ corresponds to lifetime utility of an employed agent.

The backward solution to the previous problem provides policy functions $a_n$ and $s_n$ that can be used to compute the stationary distribution of agents over unemployment spell, $\bar{\Omega}$. The stationary distribution is a fixed point of the transition law

$$\Omega_0' = \left(1 - \sum\nolimits_n \Omega_n\right)\delta$$

$$\Omega_n' = (\Omega_{n-1}(1 - p(a_{n-1}))\, \forall n < n^*,$$

$$\Omega_{n^*}' = [\Omega_{n^*-1}(1 - p(a_{n^*-1})) + \Omega_{n^*}(1 - p(a_{n^*}))].$$

In the stationary distribution, we measure the unemployment rate and the informal sector size by

$$\mu = \sum\nolimits_n \bar{\Omega}_n(1 - s_n),$$

[24] This number is obtained dividing 80 percent of the minimum wage by the income per capita.

$$\zeta = \frac{\varpi\chi}{\varpi\chi + (1 - \sum_n \bar{\Omega}_n)\omega},$$

where $\chi = \sum_n \bar{\Omega}_n s_n$ is the average participation in the hidden labor market.

In the figures, a "typical" unemployed individual is chosen as one who entered into the job market 10 years ago. Individuals enter the market with promised utility that solves $V^u(U) = 0$. Before setting it equal to zero, the planner's surplus function was adjusted to account for administrative costs. According to data from the "Ministerio de Trabajo y Asuntos Sociales", available at www.mtas.es, just above 10 percent of the expenditures on social assistance went to administrative costs during 1999.

# Notation

- An individual's labor status, $m_t$, is equal to $u_t$ if the individual is unemployed and to $e_t$ otherwise.
- The employment history of an individual up to time $t$ is represented by $h^t = (m_0, ..., m_t)$.
- The wage in the secondary or hidden labor market is represented by $\varpi$.
- Effort in the secondary or hidden labor market is represented by $s$.
- Individuals employed in the formal sector—employed individuals—must exert a constant work effort $o$, and their productivity is $\omega$.
- Individuals employed in the formal sector—employed individuals—receive wages (after taxes) $w$.
- Employment relationships end exogenously with probability $\delta$.
- Search effort during unemployment is represented by $a$.
- The job-finding probability is represented by the function $p$.
- Total effort is represented by $x$.
- The function $u$ reflects the utility from consumption.
- The function $v$ measures the cost of effort exerted either searching for a job, participating in the hidden labor market or working in the formal sector.
- The parameter $\beta \in (0, 1)$ represents the discount factor.
- The interest rate is constant at $r = 1/\beta - 1$.
- A contract or unemployment insurance system is represented by $\mathcal{W}$.
- The value of unemployment payments is represented by $b$.
- The set of direct instruments available to a planner is represented by $\mathcal{I} = \{...\}$.

- The set of prescriptions for non-observable individuals' actions that a planner decides on is represented by $\mathcal{P} = \{...\}$.
- The set of feasible contracts is represented by $\mathbf{A}$.
- A individual's consumption is represented by $c$.
- The value of a planner's payoffs is represented by $y$.
- The term $\mathcal{U}_t(\mathcal{W}; h^t)$ represents the individuals' *promised utility* associated with the contract $\mathcal{W}$ from a particular history $h^t$.
- The term $\mathcal{V}_t(\mathcal{W}; h^t)$ represents the planner's *continuation value* associated with a contract $\mathcal{W}; h^t$.
- The lower bound for $U$ is represented by $\underline{U}$.
- The promised utility provided by the constrained efficient contract is represented by $\mathcal{W}^*$.
- The term $\mathbf{C}(U; h^0)$ represents the set of incentive-compatible contracts.
- The value $\overline{u}$ represents the upper bound of the function $u$.
- The *set of promised utilities* lower than $\overline{U}$ such that $\mathbf{C}(U; h^0) \neq \emptyset$ when $h^0 = m$ is represented by $\mathbf{P}_m$.
- The set of promised utilities that the contract can deliver is represented by $\mathbf{P}$.
- A recursive contract, or unemployment insurance system, is represented by $\mathcal{W}^R$.
- The set of one-period feasible actions is represented by $\mathbf{E}$.
- The set of one-period best responses of individuals, taking as given unemployment payments and promised utility, is represented by $\mathbf{M}$.
- Promised utility for unemployed individuals getting a job next period is represented by $U^e$.
- Promised utility for unemployed individuals remaining unemployed next period is represented by $U^u$.
- The value function defining lifetime benefits for a principal offering the contract to an unemployed individual is represented by $V^u$.
- The value function defining lifetime benefits for a principal offering the contract to an employed individual is represented by $V^e$.
- The upper bound for total effort is represented by $\vartheta$.
- The term $\mathbf{M}^{foc}$ represents the set of all $(a, s) \in \mathbf{E}$ such that the constraints ICC1-ICC2 are satisfied.
- The function $f$ is defined by $f(a, s) \equiv u(b + s\varpi) - v(a) + \beta(p(a)U^e + (1 - p(a))U^u)$.
- The derivative of $V^x$ with $x \in \{e, u\}$ is represented by $V'^x$.
- Promised utility for employed individuals keeping the job next period is represented by $\tilde{U}^e$.
- Promised utility for employed individuals losing the job next period is represented by $\tilde{U}^u$.
- The multiplier associated with ICC1 is represented by $\mu_1$.
- The multiplier associated with ICC2 is represented by $\mu_2$.
- The multiplier associated with the non-negativity constraint for $s$ is represented by $\phi$.

- The multiplier associated with PKC is represented by $\lambda$.
- The multipliers associated with constraints FCU for $U^u$ and $U^e$, are $\xi^u$ and $\xi^e$, respectively.
- The term $t^*$ represents a period $t$ such that $U^u_{t^*} = \underline{U}$.
- The lower bound for consumption—under linear v—is represented by $\underline{c}$.
- The level of consumption prescribed by the contact—under linear v— once the lower bound for promised utility is reached is represented by $\overline{c}$.