Searching for Hysteresis[*]

Luca Benati
University of Bern[†]

Thomas A. Lubik
Federal Reserve Bank of Richmond[‡]

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ONLINE APPENDIX

[*]The views expressed in this paper do not necessarily reflect those of the Federal Reserve Bank of Richmond or of the Federal Reserve System.

[†]Department of Economics, University of Bern, Schanzeneckstrasse 1, CH-3001, Bern, Switzerland. Email: luca.benati@vwi.unibe.ch

[‡]Research Department, Federal Reserve Bank of Richmond, P.O. Box 27622, Richmond, VA 23261. Tel.: +1-804-697-8246. Email: thomas.lubik@rich.frb.org.

A Data

We collect quarterly data that are seasonally adjusted with the exception of the interest rates. Data that are sampled at higher frequency are aggregated to quarterly frequency. All data sources are publicly available, either from official sources, data providers or freely available on the web.

A.1 United States

The quarterly seasonally adjusted series for real GDP ("GDPC1: Real Gross Domestic Product, Billions of Chained 2012 Dollars, Quarterly, Seasonally Adjusted Annual Rate") and the GDP deflator ("GDPCTPI: Gross Domestic Product: Chain-type Price Index") are from the Bureau of Economic Analysis. We compute inflation as the log-difference of the GDP deflator.

The seasonally adjusted series for real chain-weighted investment and real chain-weighted consumption of non-durables and services are computed based on the data found in Tables 1.1.6, 1.1.6B, 1.1.6C, and 1.1.6D of the United States National Income and Product Accounts. Real consumption includes non-durables and services, while real investment is computed by chain-weighting the respective series for durable goods; private investment in structures, equipment, and residential investment; Federal national defense and non-defense gross investment; and state and local gross investment. Both series are available at quarterly frequency.

Quarterly seasonally adjusted series for hours worked by all persons in the nonfarm business sector ("HOANBS: Nonfarm Business Sector: Hours of All Persons, Index 2009=100"), and employment in the nonfarm business sector ("PRS85006013: Nonfarm Business Sector: Employment, Index 2009=100") are from the Bureau of Labor Statistics. A quarterly seasonally unadjusted series for population ("B230RC0Q173SBEA: Population, thousands, quarterly, not seasonally adjusted") is obtained from the Bureau of Economic Analysis.

Monthly series for the civilian labor force ("CLF16OV"), the civilian unemployment rate ("UNRATE"), and the unemployment level ("UNEMPLOY") are from the Bureau of Labor Statistics publication *Employment Situation*. The series are converted to quarterly frequency by taking averages within the quarter.

We obtain a monthly series for the 10-year government bond yield ("GS10: 10-Year Treasury Constant Maturity Rate") from the Board of Governors of the Federal Reserve System. A monthly series for the Wu and Xia (2016) "shadow rate" is from Cynthia Wu's website: https://sites.google.com/view/jingcynthiawu. Both the Wu-Xia "shadow rate" and the 10-year government bond yield are converted to quarterly frequency by taking averages within the quarter.

A.2 United Kingdom

Quarterly seasonally adjusted series for nominal GDP ("YBHA: Gross Domestic Product at market prices, Current price, Seasonally adjusted £m'), real GDP ("ABMI: Gross Domestic Product, £ million at chained volume measures"), real consumption ("ABJR: Household final consumption expenditure, £ million at chained volume measures"), real investment ("NPQT: Total gross fixed capital formation, £ million at chained volume measures"), and real government expenditure ("NMRY: General Government, Final consumption expenditure, £ million at chained volume measures") are all from the U.K. Office for National Statistics. The series for the GDP deflator is computed as the ratio between nominal and real GDP.

A quarterly seasonally adjusted series for total hours worked and a quarterly seasonally unadjusted series for population are from the latest version (updated to 2016Q4) of the Ohanian and Raffo (2012) dataset.

Monthly series for a short rate ("Spliced series for Treasury bill allotment rate/discount rate at the weekly tender") and a long rate ("Spliced 10-year yield") are from "A millennium of macroeconomic data for the UK: The Bank of England's collection of historical macroeconomic and financial statistics, Version 3 - finalized 30 April 2017", which is available from the Bank of England's website. A monthly series for the Wu-Xia "shadow rate" is from Cynthia Wu's website. All original monthly series are converted to quarterly frequency by taking averages within the quarter.

A.3 Euro Area

All Euro Area data are obtained from the European Central Bank. As before, a monthly series for the Wu-Xia "shadow rate" is downloaded from Cynthia Wu's website. The shadow rate is converted to quarterly frequency by taking averages within the quarter.

B Computing the Structural Impact Matrix A_0

We compute the structural impact matrix A_0 based on the methodology proposed by Arias et al. (2018) for combining zero and sign restrictions when a researcher has to draw from the posterior distribution of the VECM's reduced-form parameters.

Specifically, let B_0^j, B_1^j, ..., B_p^j, β^j, α^j, and Ω^j be the j-th draw from the posterior distribution for the intercept, the VAR matrices, the matrix of the cointegration vectors, the matrix of the loadings, and the covariance matrix of reduced-form innovations of the VECM:

$$\Delta Y_t = B_0 + B_1 \Delta Y_{t-1} + ... + B_p \Delta Y_{t-p} + \alpha\beta' Y_{t-1} + u_t, \text{ for } j = \ 1, 2, 3, ..., J.$$

Let $P_j D_j P_j'$ be the eigenvalue-eigenvector decomposition of Ω^j. We start by computing an initial estimate of A_0^j, labelled $\tilde{A}_0^j$, as $\tilde{A}_0^j = P_j D_j^{\frac{1}{2}}$ with the corresponding matrix of long-run impacts of the structural shocks $\tilde{L}_j$.

Based on the Gibbs-sampling algorithm described in section 3.6.3 of Arias et al. (2014), we then draw Z random orthonormal matrices of dimension $N \times N$ from a uniform distribution. N is the number of series in the VECM, conditional on the zero restrictions on the long-run impacts of the structural shocks on GDP. That is, two shocks are allowed to have a non-zero long-run impact on GDP, whereas the remaining five shocks are forced to have no long-run impact.

Let Q_z^j, z = 1, 2, 3, ..., Z, be the z-th random orthonormal matrix, with $Q_z^j (Q_z^j)' = I_K$. We combine each of the Z random orthonormal matrices with the initial estimate of the long-run impact of the structural shocks, $\tilde{L}_j$, to obtain a randomly rotated long-run impact matrix, $L_j^z = \tilde{L}_j Q_j^n$. By construction, each L_j^z, $z = \ 1, 2, 3, ..., Z$, satisfies the zero long-run restrictions. We then obtain the corresponding candidate estimate of the structural impact matrix, $A_{0,j}^z$ from L_n^j.

Finally, we check whether $A_{0,j}^z$ and the associated impulse-response functions up to horizon H satisfy the sign restrictions discussed in section 2.3. Out of the Z candidate structural impact matrices we only keep those satisfying the sign restrictions for each draw j. For each draw from the posterior we consider 100 random rotation matrices. Finally, we set the number of Gibbs-sampling iterations in the algorithm to $L = 10$.[1]

C Additional Tables and Figures

[1] As pointed out by Arias et al. (2018), *"[t]here is also the question of how large L should be to obtain convergence. Experiments show that for the starting value given below, even $L\ = 1$ gives a good approximation of the desired distribution. In practice, increasing values of L can be used to determine when convergence has occurred."* Our own experience with the algorithm confirms this. Although we set $L\ = 10$, convergence is typically achieved at most at the second iteration.

Table A.1 Bootstrapped *p*-values for Elliot *et al.* (1996) unit root tests in levels[a]

	United States, 1954Q3-2019Q3				Euro area, 1970Q1-2017Q4				United Kingdom, 1971Q1-2016Q4			
	p=1	p=2	p=3	p=4	p=1	p=2	p=3	p=4	p=1	p=2	p=3	p=4
	I: Tests with an intercept and a time trend											
Log real GDP	0.385	0.600	0.830	0.827	0.035	0.243	0.346	0.390	0.707	0.508	0.492	0.412
Log real consumption	0.251	0.711	0.660	0.872	0.002	0.019	0.094	0.365	0.942	0.896	0.762	0.729
Log real investment	0.619	0.371	0.352	0.365	0.804	0.711	0.544	0.308	0.037	0.019	0.031	0.051
Log GDP deflator	0.966	0.964	0.917	0.870	0.018	0.233	0.390	0.116	0.998	0.993	0.983	0.971
Log total hours	0.605	0.523	0.642	0.694					0.129	0.457	0.634	0.727
Log average hours	0.890	0.842	0.794	0.817					0.861	0.773	0.538	0.351
Log population	0.001	0.301	0.099	0.802					0.781	0.797	0.681	0.690
Log participation	0.999	0.998	0.992	0.995					0.684	0.384	0.169	0.182
Log unemployment	0.677	0.175	0.237	0.382								
Log labor force					0.977	0.933	0.805	0.739				
Log employment					0.375	0.174	0.190	0.248				
	II: Tests with an intercept and no time trend											
Short rate	0.274	0.244	0.247	0.168	0.904	0.840	0.862	0.895	0.796	0.680	0.693	0.712
Long rate	0.659	0.676	0.630	0.650	0.905	0.896	0.901	0.932	0.839	0.886	0.894	0.898
Inflation	0.023	0.102	0.202	0.218	0.193	0.441	0.685	0.568	0.000	0.007	0.043	0.079
Unemployment rate	0.354	0.008	0.019	0.042	0.162	0.113	0.177	0.164	0.400	0.319	0.269	0.348
Log participation	0.312	0.260	0.257	0.399					0.482	0.355	0.158	0.137

[a] Based on 10,000 bootstrap replications of estimated ARIMA processes.

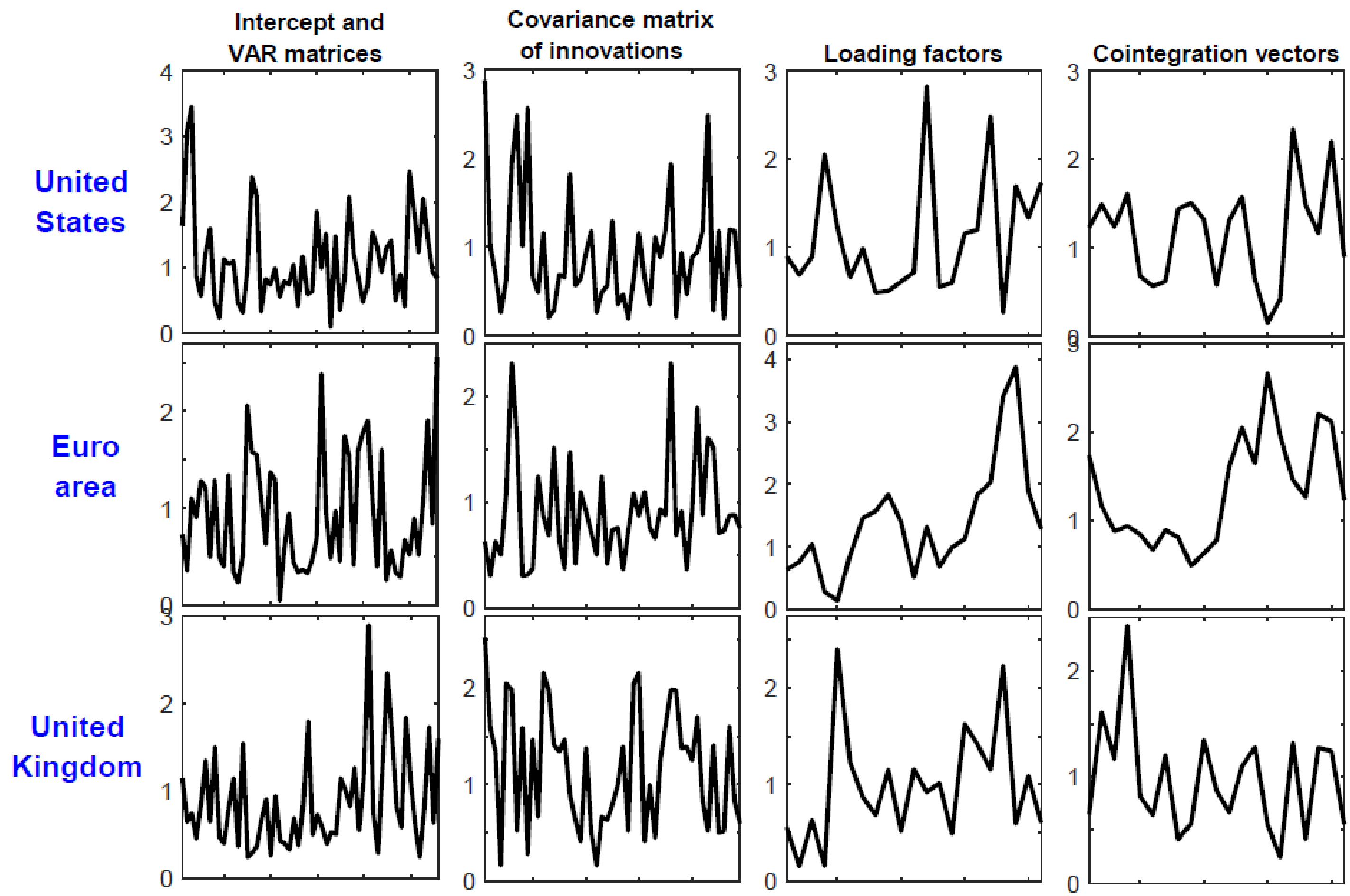

Figure A.I.1 Checking the convergence to the ergodic distribution: inefficiency factors of the parameters' draws (for models imposing the 5% restriction on the maximum deviation of the transitory component of GDP from the official output gap estimates)

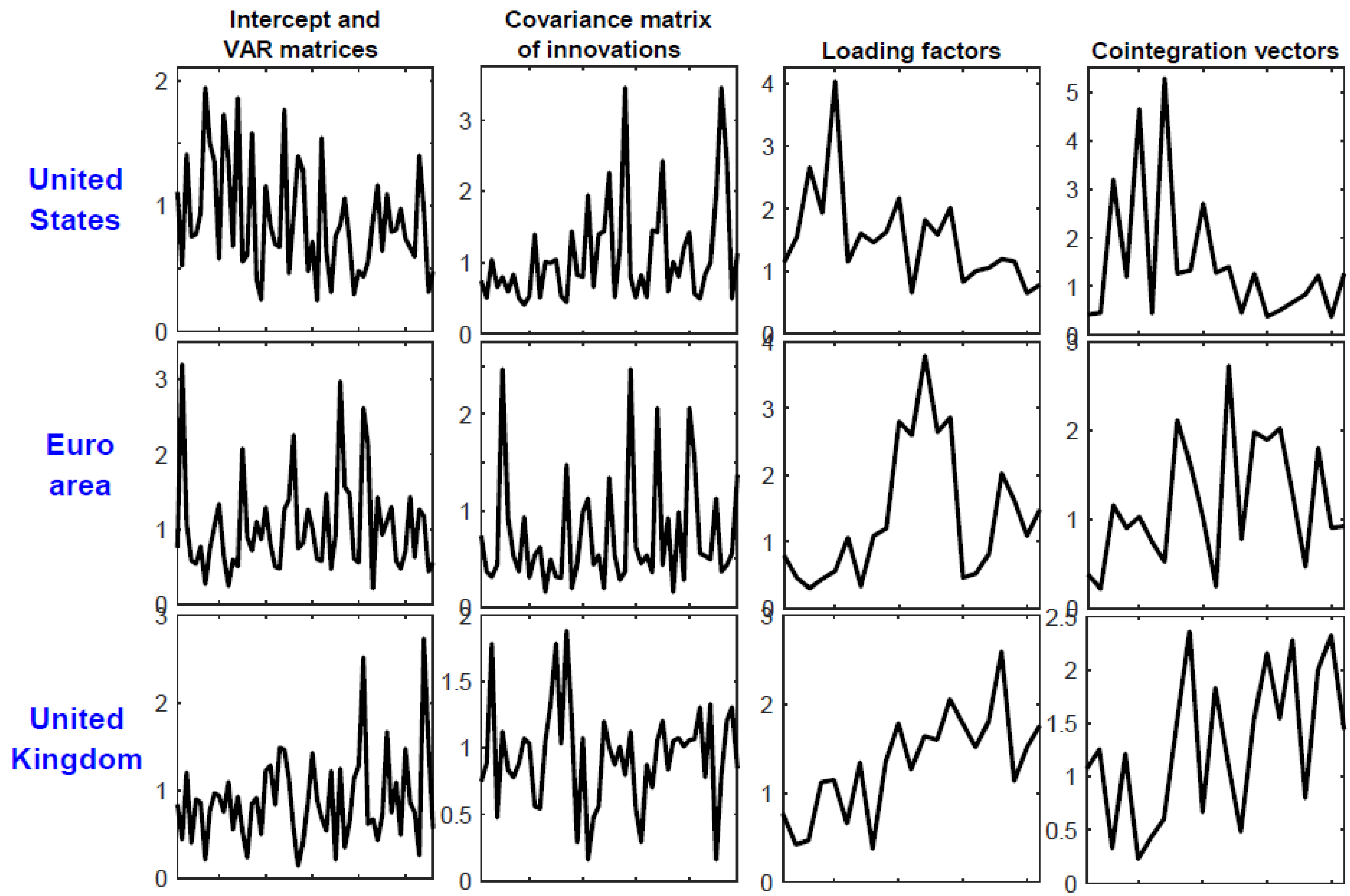

Figure A.I.2 Checking the convergence to the ergodic distribution: inefficiency factors of the parameters' draws (for models not imposing the 5% restriction on the maximum deviation of the transitory component of GDP from the official output gap estimates)

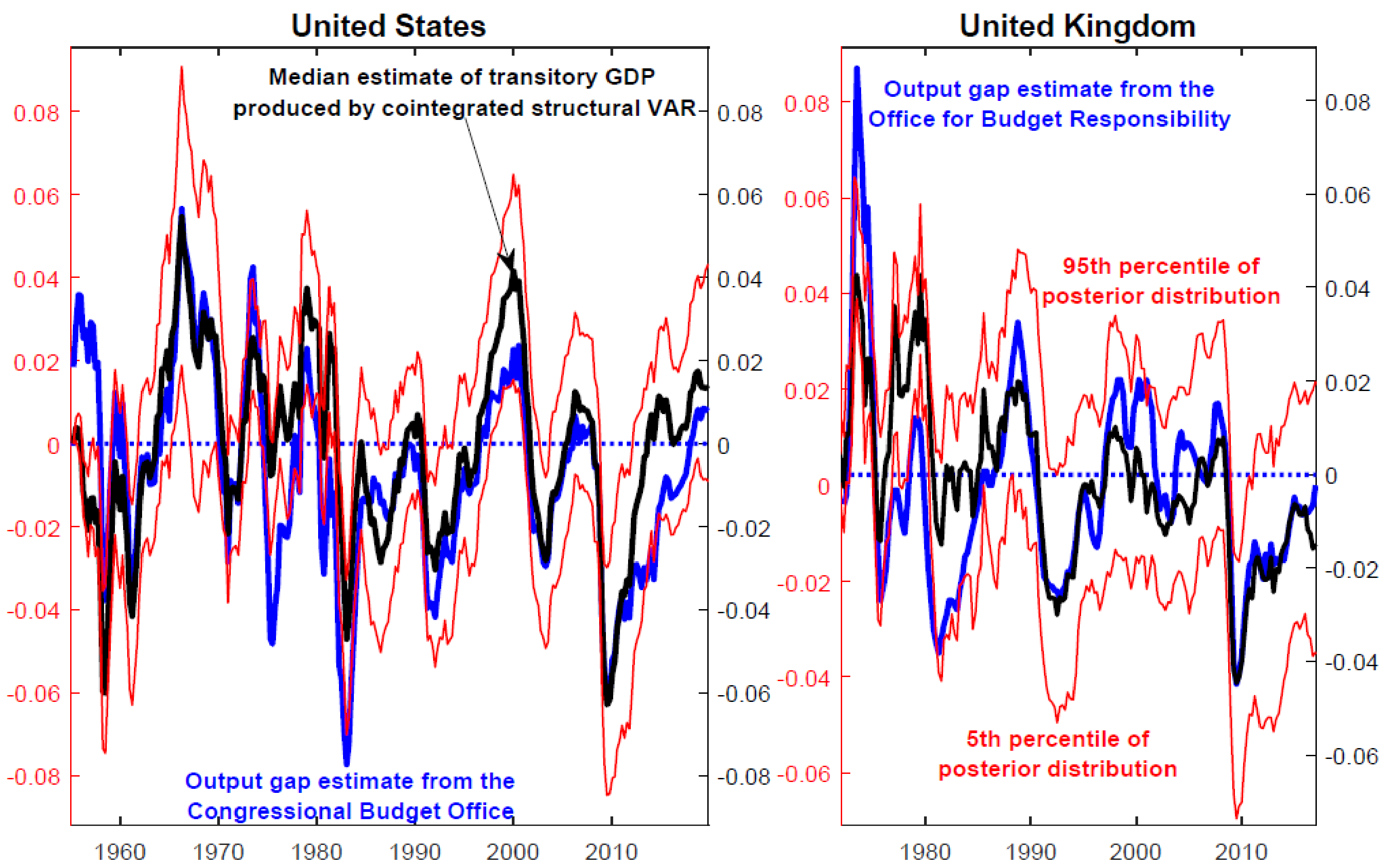

Figure A.I.3 The transitory component of GDP produced by the Bayesian cointegrated SVARs (imposing the 5% restriction on the maximum deviation of the transitory component of GDP from the official output gap estimates)

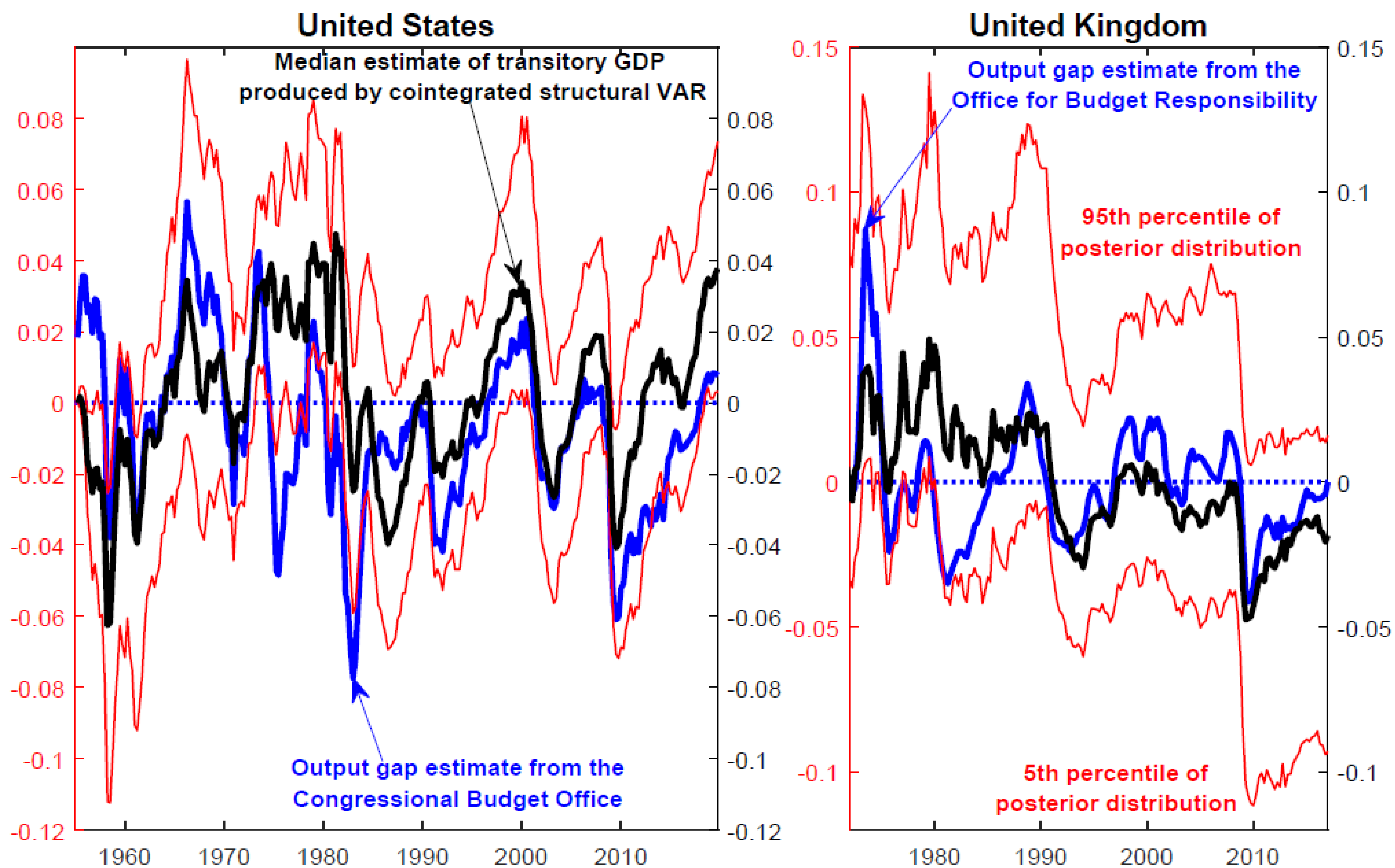

Figure A.I.4 The transitory component of GDP produced by the Bayesian cointegrated SVARs (not imposing the 5% restriction on the maximum deviation of the transitory component of GDP from the official output gap estimates)

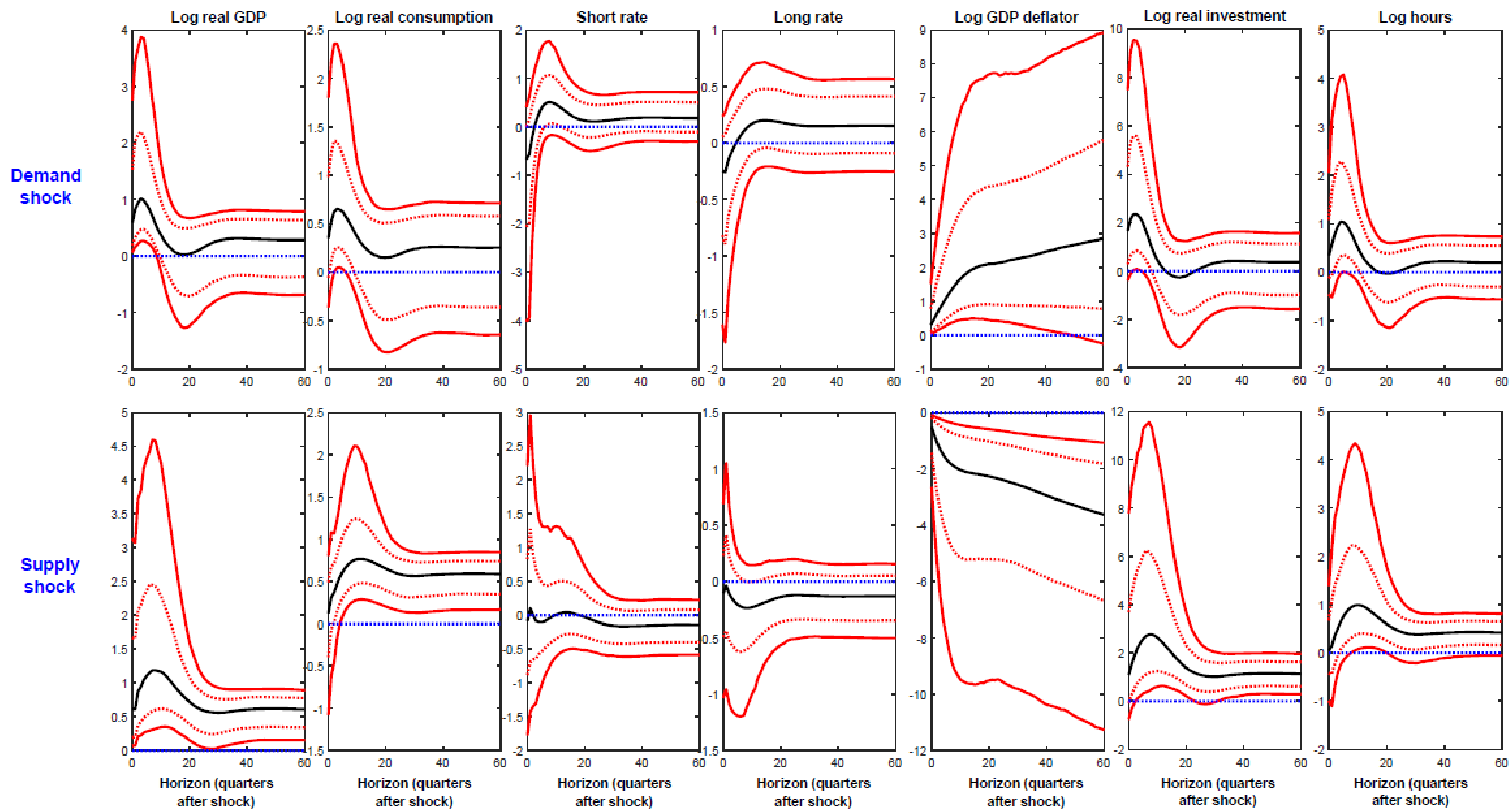

Figure A.I.5 Bayesian evidence for the U.S. for the ‘non-ZLB’ sample: impulse-response functions to demand and supply shocks, without imposing the restriction that the signs of the long-run impacts of demand shocks on GDP are the same for all draws (medians, and 16-84 and 5-95 percentiles)

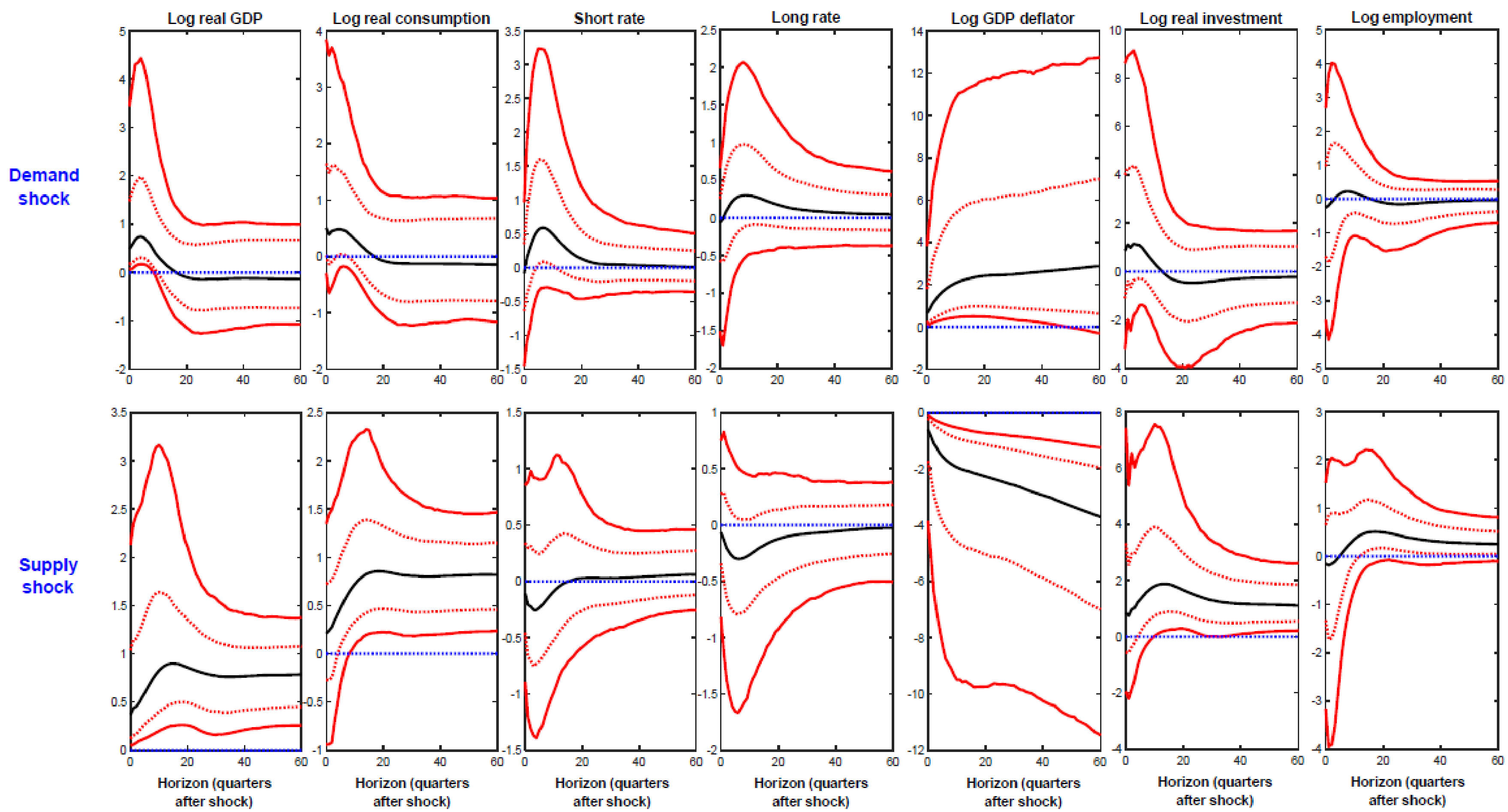

Figure A.I.6 Bayesian evidence for the Euro area for the 'non-ZLB' sample: impulse-response functions to demand and supply shocks, without imposing the restriction that the signs of the long-run impacts of demand shocks on GDP are the same for all draws (medians, and 16-84 and 5-95 percentiles)

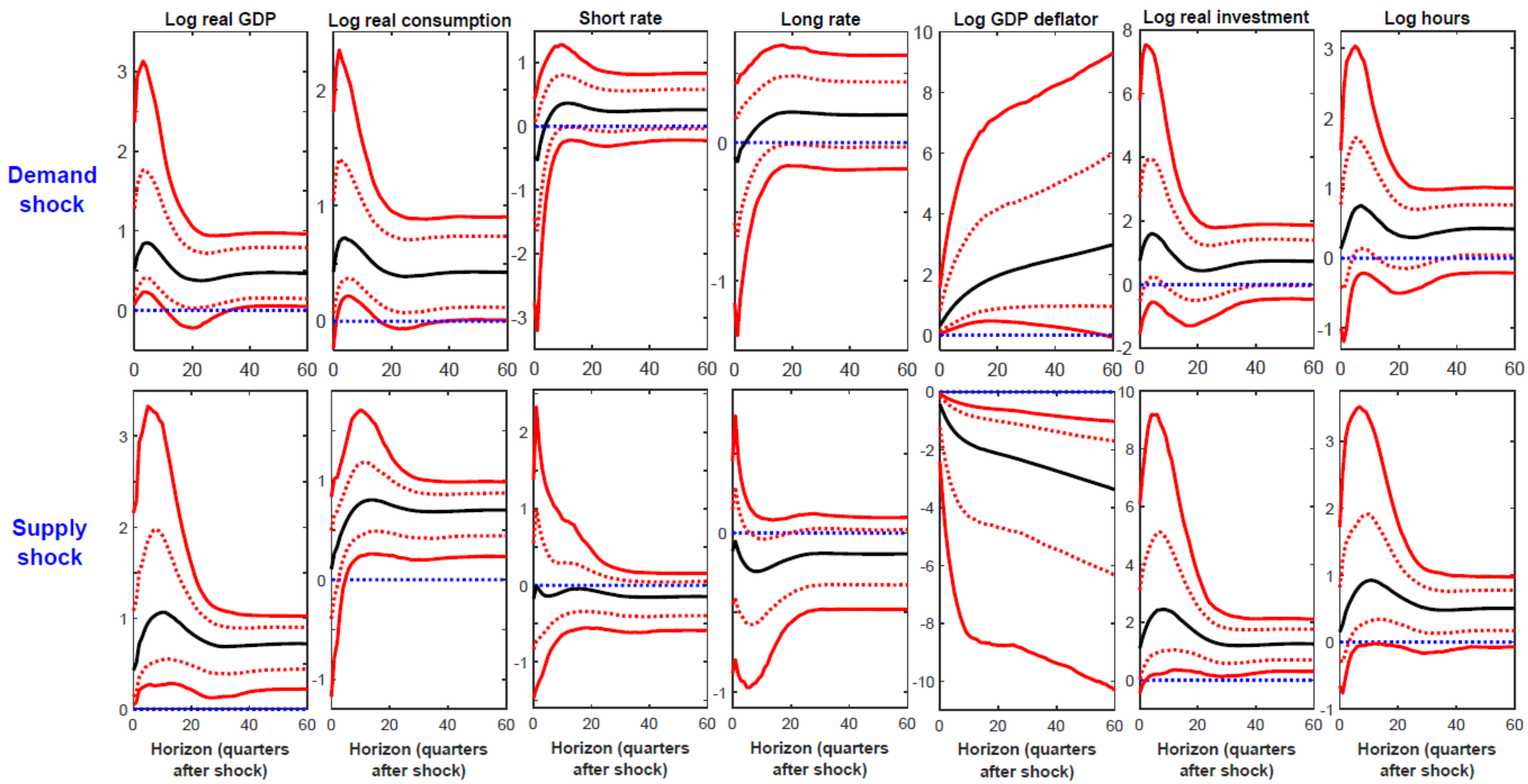

Figure A.I.7 Bayesian evidence for the U.S.: impulse-response functions to demand and supply shocks, imposing the restriction that the signs of the long-run impacts of demand shocks on GDP are the same for all draws (medians, and 16-84 and 5-95 percentiles)

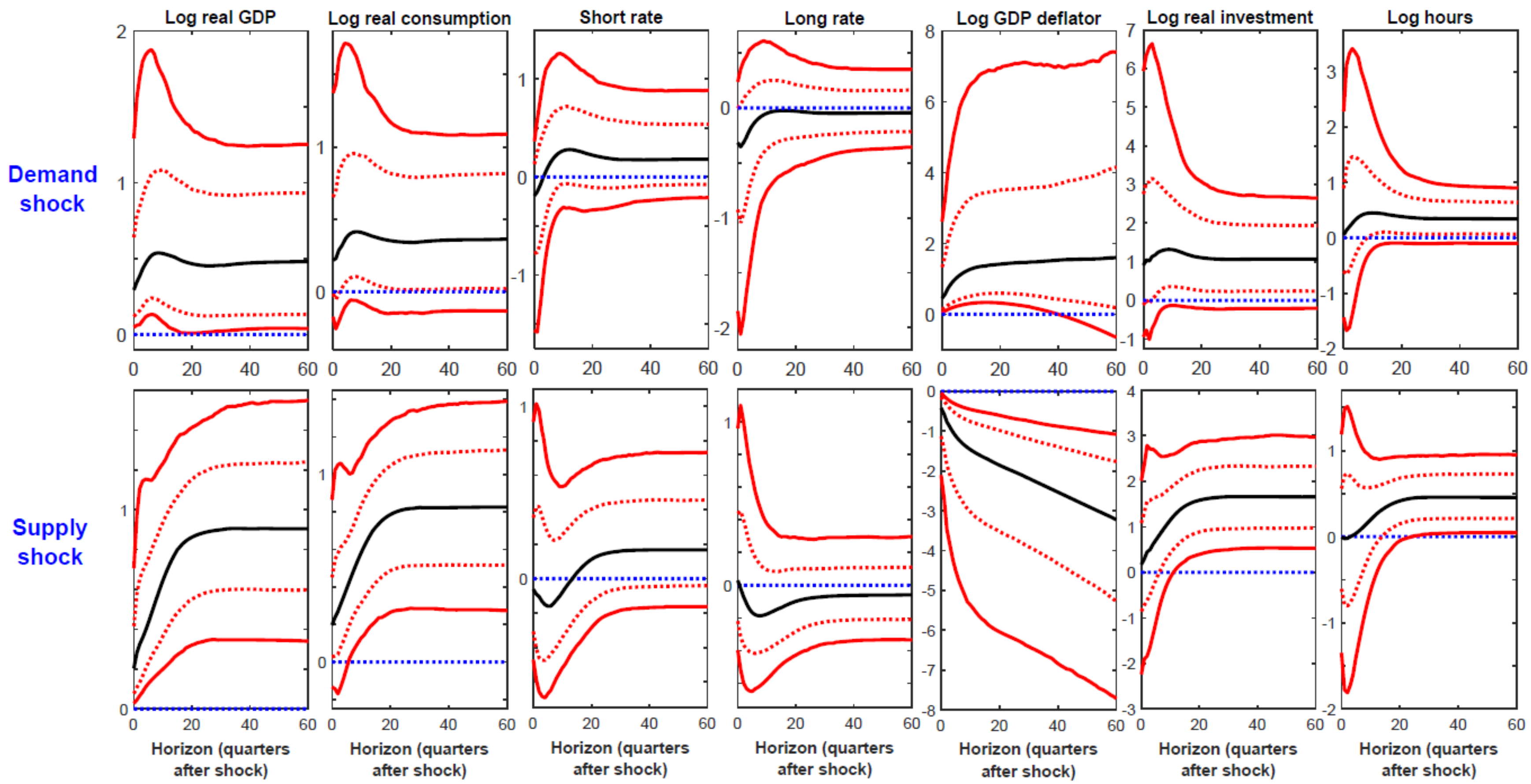

Figure A.I.8 Bayesian evidence for the Euro area: impulse-response functions to demand and supply shocks, imposing the restriction that the signs of the long-run impacts of demand shocks on GDP are the same for all draws (medians, and 16-84 and 5-95 percentiles)

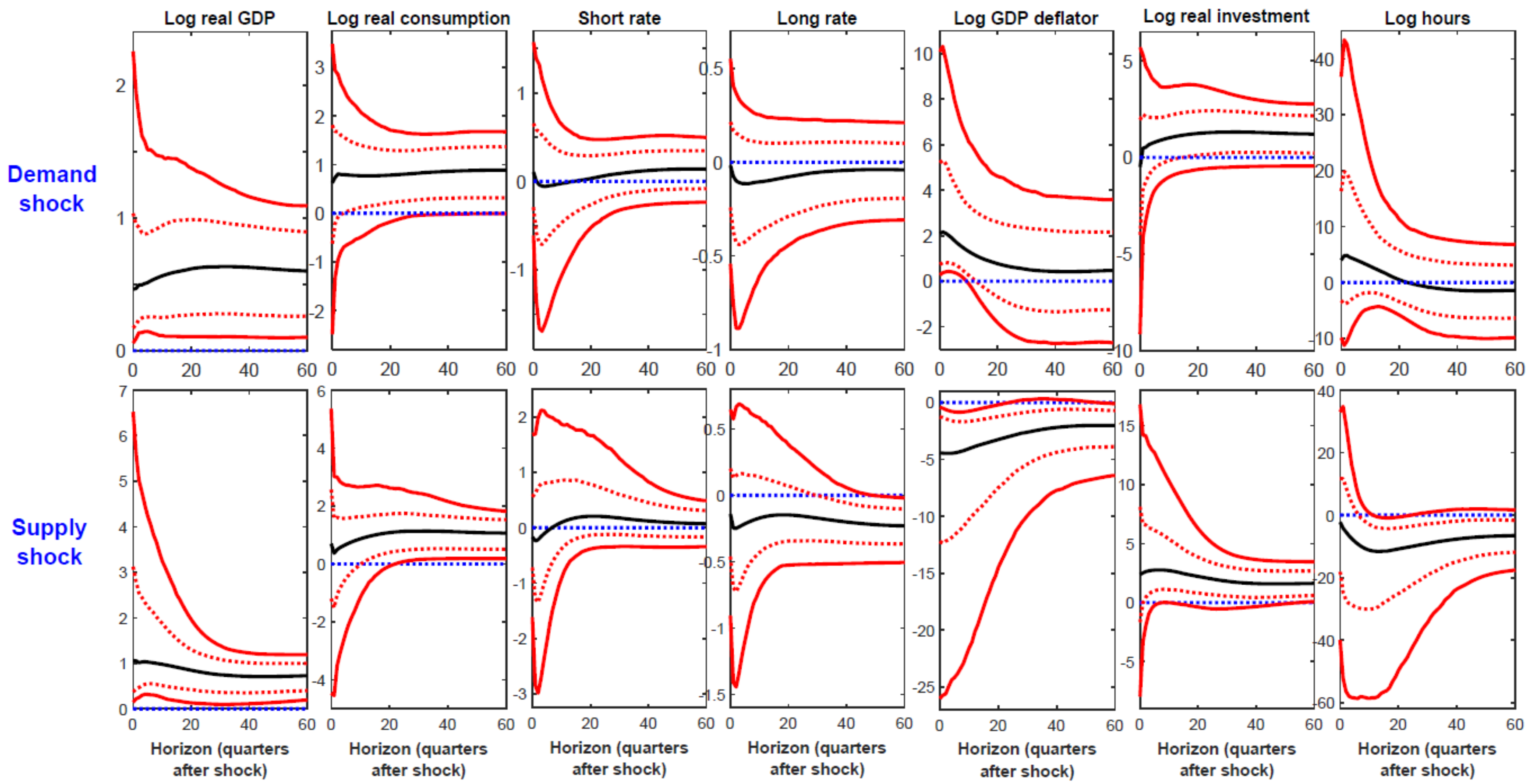

Figure A.I.9 Bayesian evidence for the U.K.: impulse-response functions to demand and supply shocks, imposing the restriction that the signs of the long-run impacts of demand shocks on GDP are the same for all draws (medians, and 16-84 and 5-95 percentiles)

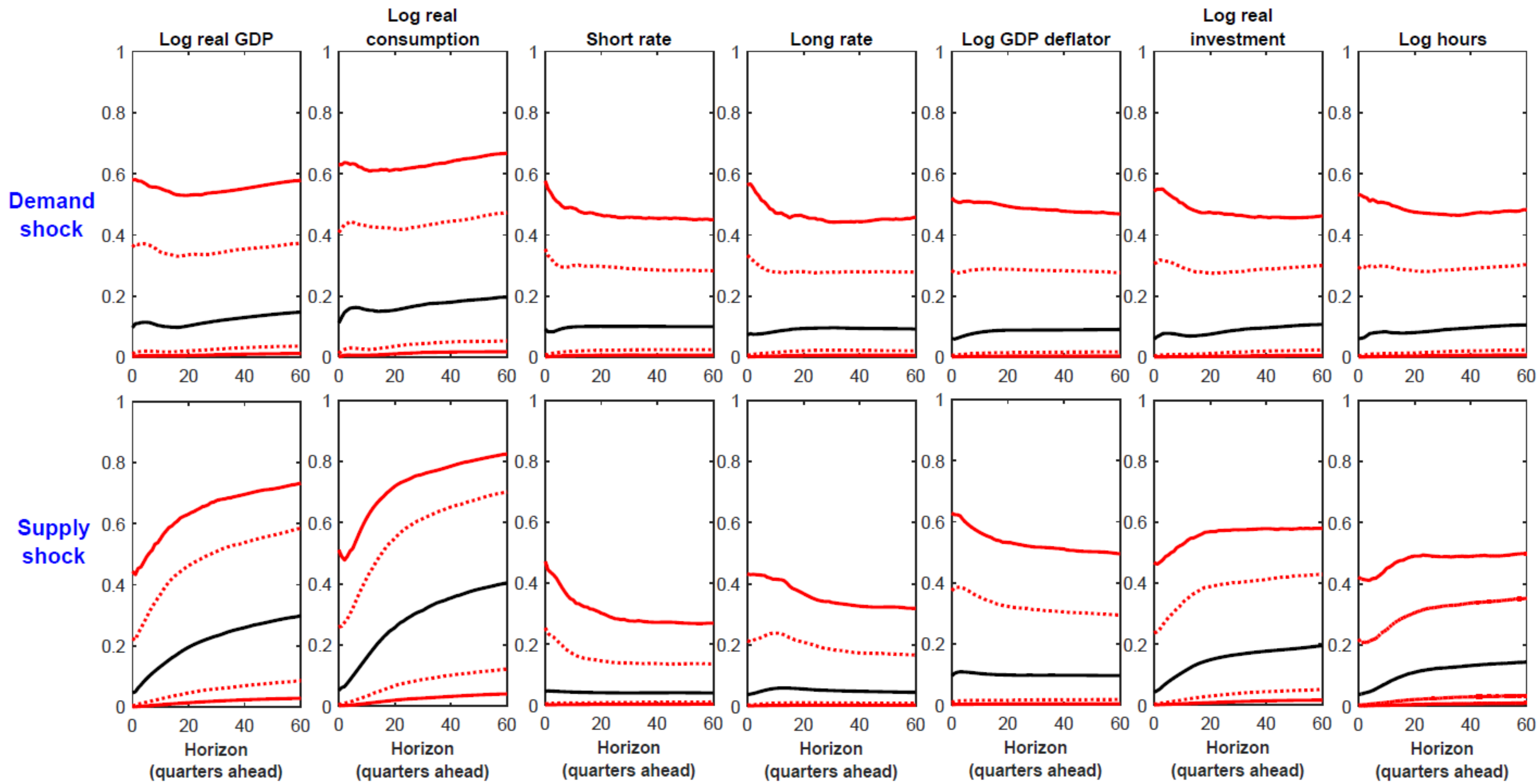

Figure A.I.10 Bayesian evidence for the U.S.: fractions of forecast error variance explained by demand and supply shocks, without imposing the restriction that the signs of the long-run impacts of demand shocks on GDP are the same for all draws (medians, and 16-84 and 5-95 percentiles)

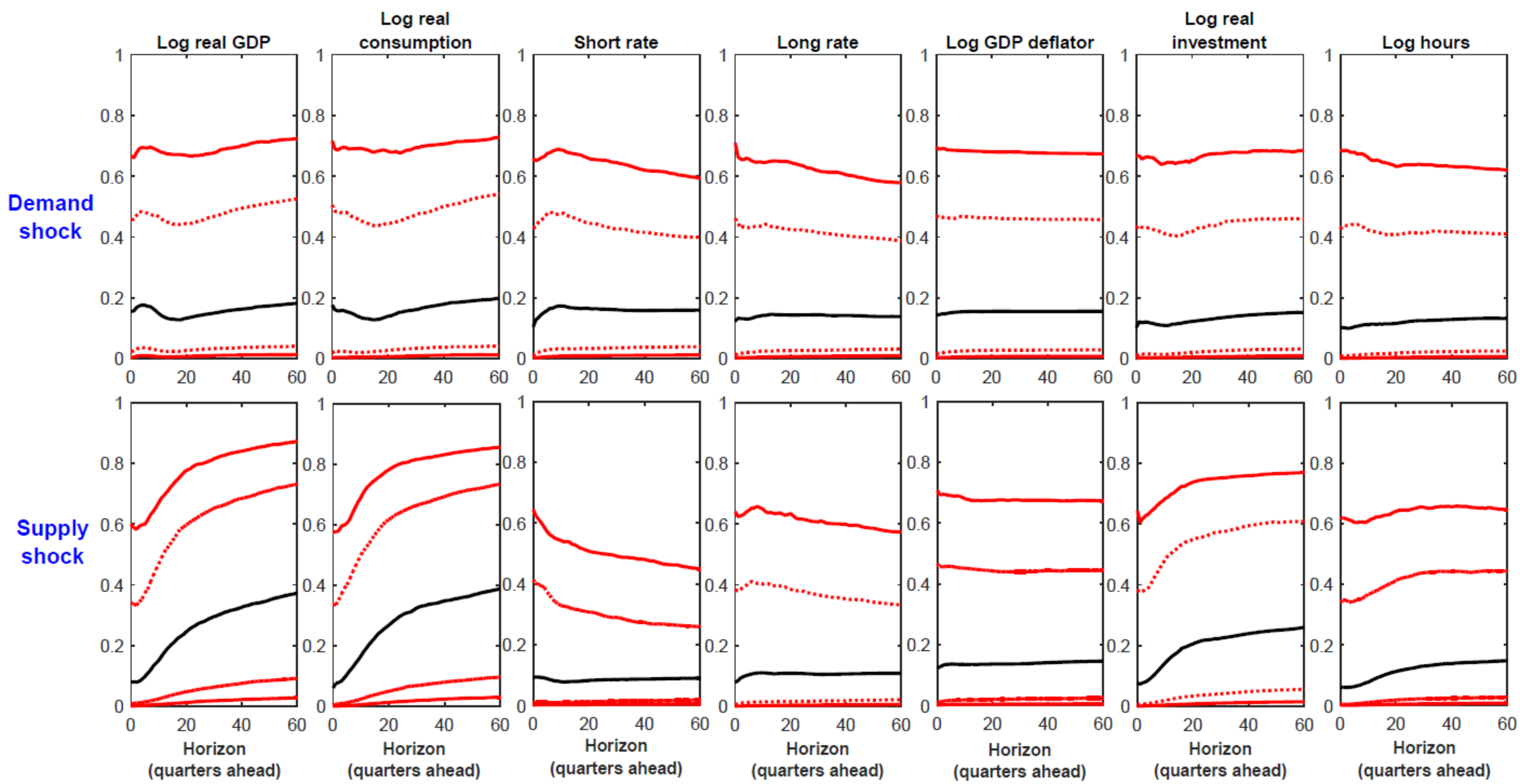

Figure A.I.11 Bayesian evidence for the Euro area: fractions of forecast error variance explained by demand and supply shocks, without imposing the restriction that the signs of the long-run impacts of demand shocks on GDP are the same for all draws (medians, and 16-84 and 5-95 percentiles)

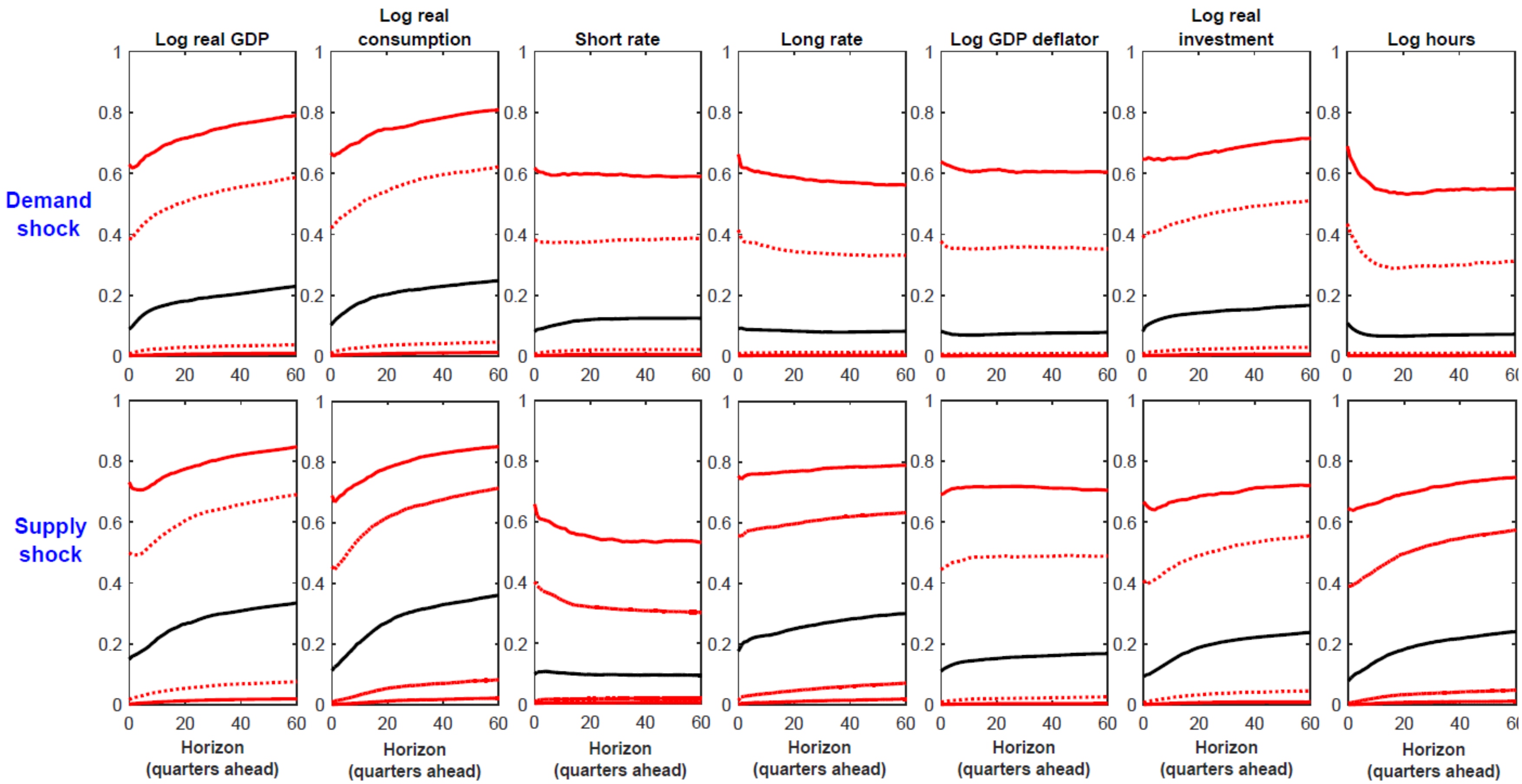

Figure A.I.12 Bayesian evidence for the U.K.: fractions of forecast error variance explained by demand and supply shocks, without imposing the restriction that the signs of the long-run impacts of demand shocks on GDP are the same for all draws (medians, and 16-84 and 5-95 percentiles)

References

[1] Arias, Jonas E., Juan F. Rubio-Ramírez, and Dan F. Waggoner (2018): "Inference Based on SVARs Identified with Sign and Zero Restrictions: Theory and Applications". *Econometrica*, pp. 685-720.

[2] Ohanian, Lee E., and Andrea Raffo (2012): "Aggregate hours worked in OECD countries: New measurement and implications for business cycles". *Journal of Monetary Economics*, 59(1), pp. 40-56.

[3] Wu, Jing Cynthia, and Fan Dora Xia (2016): "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound". *Journal of Money, Credit, and Banking*, 48(2-3), pp. 253-291.

[4] Wu, Jing Cynthia, and Fan Dora Xia (2017): "Time Varying Lower Bound of Interest Rates in Europe". Manuscript.